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Equilibrium Analysis of Coexisting IEEE 802.11e Wireless LANs

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***Abstract*—A model for the coexistence problem of overlapping IEEE 802.11e wireless networks is discussed in this paper. In a competition scenario of overlapping 802.11e networks, quality of service cannot be guaranteed by the 802.11e protocol. For this reason, a stage-based game structure is introduced here. Wireless networks participate in the game as players. The players are gaining in each stage a so-called utility, which is a summarizing value for the achieved quality of service. The interaction of the players within a single stage is analyzed in detail, under consideration of Nash Equilibria and Pareto efficiency.**

Keywords—IEEE 802.11e; Coexistence of 802.11e WLANs; Single Stage Games; Multi Stage Games; QoS as Utility.

I. INTRODUCTION

The *Institute of Electrical and Electronics Engineers, Inc. (IEEE)* develops the *IEEE 802.11e (802.11e)* as an extension of the IEEE 802.11 *Wireless Local Area Network (WLAN)* standard [1]-[3]. 802.11e is defined for the provisioning of priorities in medium access, to enable wireless LANs to achieve data throughput and delay constraints, hence, to support *Quality-of-Service (QoS)*. By applying 802.11e, wireless networks support multimedia and Internet applications that require QoS.

According to the 802.11e standard, an access point together with its associated stations form a *QoS supporting Basic Service Set (QBSS)*. Because WLANs operate mainly in unlicensed frequency bands, QBSSs often have to operate in problematic situations. QBSSs have to share radio resources among when multiple QBSSs operate at the same channel, time and location. Such coexistence problems are not addressed in detail in the enhanced standard 802.11e. Specifically, if all QBSSs access the medium with highest priority, an individual control over the medium is not feasible.

This paper presents the equilibrium analysis developed in [4]-[6]. The analysis uses an analytical abstraction of the coexistence problem as discussed in [7]. This analytical approach is extended in Section II of this paper, where a dynamic stage-based game structure is defined to study the coexistence of QBSSs. This structure comprises a set of two decision making entities referred to as players [8]. Players choose actions in each stage of the game, the *Single Stage Game (SSG)*. The duration of a SSG is defined by a superframe, which is a time interval defined by the 802.11 standard. Repeated SSGs form a *Multi Stage Game (MSG)*. The players' QoS parameters, the resulting players' action and a thereon based definition of an abstract utility function are introduced in Section III. The players' outcome of the SSG namely the payoff is analyzed in detail. The concepts of a *Nash Equilibrium (NE)* and Pareto efficiency are used to illuminate the potential of game solutions in Section V. In extending the scope to MSGs in Section VI the payoff maximizing players are able to improve their outcome through dynamic interaction. Therefore, behaviors are defined and evaluated with the help of simulation. The paper ends with a conclusion in Section VII.

II. STAGE-BASED GAME STRUCTURE

An SSG consists of three phases: (1) the players decide about their action, by means of demanded allocation point of times and lengths. (2) The allocation process: The players' allocations, also referred to as *Transmission Opportunities (TXOPs)*, delay each other or may collide and therefore the observed allocation point of times may differ from the demanded times. After the competition, the players calculate the outcomes of the SSG with the help of a utility function (3).

III. QUALITY OF SERVICE AS UTILITY

We define three abstract and normalized representations of the QoS parameters: (1) the throughput $\Theta \in [0..1]$, (2) the delay $\Delta \in [0..0.1]$ and (3) the delay variation $\Xi \in [0..0.1]$. This delay variation is not considered here. The throughput $\Theta^i(n)$ represents the share of capacity a player ***i*** demands in stage *n* of the game:

$$\Theta^i(n) = \frac{1}{SFDUR(n)} \sum_{l=1}^{L^i(n)} d_l^i(n) \tag{1}$$

$L^i(n)$ is the number of allocated TXOPs per superframe *n* and $SFDUR(n)$ the duration of this superframe in *ms*, typically with a length of *200 ms*. The parameter $d_l^i(n)$ describes the duration of the TXOP *l*, *l=1...L*, in *ms*, of player *i* in stage *n*.

The TXOP delay $\Delta^i(n)$ specifies the maximum delay that the player tolerates at superframe *n*. In particular, this delay describes the expected maximum delay between two TXOPs due to the interrupted TXOP allocations:

$$\Delta^i(n) = \frac{1}{SFDUR(n)} max\left[D_l^i(n)\right]_{l=1..L(n)-1} \quad (2)$$

$D_l^i(n)$ is the time between the starting points of the two successive TXOPs l and $l+1$ of player i in superframe n, again measured in *ms* .

Each player i calculates the outcomes with three different sets of QoS parameters: the “required” (*req*), “demanded” (*dem*) and “observed” (*obs*) QoS parameters. Fig. 1 illustrates the interdependencies of these parameters in the context of a repeated SSG.

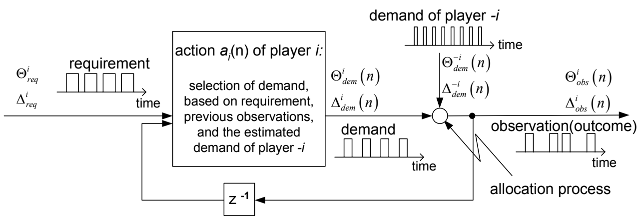

Fig. 1: The different QoS parameters of player *i* in an SSG.

Player *i*’s required QoS parameters Θ^i_{req} and Δ^i_{req} are defined through the QoS traffic which the player is trying to support. Before each SSG the players decide about their demanded allocations, i.e. actions, leading to the demanded QoS parameters Θ^i_{dem} and Δ^i_{dem}. They are changed by the player from stage to stage and determine the allocation point of times and lengths within a superframe, resp. an SSG. In general, a player observes less and delayed TXOPs through the competitive allocation process. This leads to the observed QoS parameters Θ^i_{obs} and Δ^i_{obs} as outcome of the SSG.

Based on the introduced QoS parameters the actual operation of a player is defined, in the game called action $a^i(n)$ of player i in stage n of an MSG. Each player decides at the beginning of each stage, i.e. SSG, about its action. We assume a simplified game of $N=2$ players and thus the opponent is referred to as player $-i$ in the following. An action of player i consists of the two demanded QoS parameters and is defined as

$$\begin{pmatrix}\Theta^i_{dem}(n)\\ \Delta^i_{dem}(n)\end{pmatrix} := \left(\begin{pmatrix}\Theta^i_{req}\\ \Delta^i_{req}\end{pmatrix}, \begin{pmatrix}\tilde{\Theta}^{-i}_{dem}(n-1)\\ \tilde{\Delta}^{-i}_{dem}(n-1)\end{pmatrix}, H^n\right) \rightarrow a^i(n). \quad (3)$$

An action depends on the players’ QoS requirement $(\Theta^i_{req}, \Delta^i_{req})$ and the expected action, i.e. demand, of the opponent $(\tilde{\Theta}^{-i}_{dem}, \tilde{\Delta}^{-i}_{dem})$. The superscript indicates that the action of the opponent is not known to the player but estimated from the observed history of the game. Therefore, only the opponent’s action of the previous stage can be considered. In addition to this the history H^n of own observed QoS parameters $(\Theta^i_{obs}, \Delta^i_{obs})$ up to the previous stage $n-1$ is evaluated.

The utility represents the supported QoS of a player and depends consequently on the two above introduced QoS parameters. The definition of the utility function considers all characteristics of the QoS under consideration of the requirements. More precisely defines the utility $U^i(n) \in \mathbb{R}_0^+$ what player i gains from a specific action $a^i(n)$ in stage n. This dependency on the stage n is left away in the following.

The utility of player i depends of the two normalized utility terms U^i_Θ and U^i_Δ. They represent the observed share of capacity and point of times of resource allocation. Consequently the overall utility is given through

$$U^i = U^i_\Theta\left(\Theta^i_{dem}, \Theta^i_{obs}, \Theta^i_{req}\right) \cdot U^i_\Delta\left(\Delta^i_{obs}, \Delta^i_{req}\right) \quad (4)$$

where U^i is a non negative real number. All utility terms have values between 0 and 1. The utility function of the gained throughput U^i_Θ is defined as

$$U^i_\Theta\left(\Theta^i_{dem}, \Theta^i_{obs}, \Theta^i_{req}\right) := \left(1 - \frac{1}{1 + u \cdot \left(\Theta^i_{obs} - \Theta^i_{req} + \Theta^i_{tolerance}\right)}\right), \quad (5)$$
$$\cdot\left(1 + v \cdot \left(\Theta^i_{req} - \Theta^i_{dem}\right)\right)$$

if $\Theta^i_{obs} \geq \Theta^i_{req} - \Theta^i_{tolerance}$ else $U^i_\Theta\left(\Theta^i_{dem}, \Theta^i_{obs}, \Theta^i_{req}\right) := 0$. The parameters u and v in Equation (5) define the elasticity of the utility function. The tolerable deviation of the share of capacity is given as

$$\Theta^i_{tolerance} = \frac{\sqrt{v^2 + u \cdot v} - v}{u \cdot v}, \quad u, v \in \mathbb{R}^+, \quad u, v > 0 \ .$$

The two shaping parameters $u, v \in \mathbb{R}^+, u, v > 0$ are assumed to be constant and known to the players through the complete game. To force the player not to allocate much of the medium the shaping factor v reduces the observed utility for high values of Θ^i_{dem} to the benefit of the third player. The parameter u appends some kind of elasticity to the game structure: depending on this parameter the player may strictly need its requirement or it is satisfied with less adequate observations.

The second utility term U^i_Δ, which is related to the period of resource allocations, depends on the same parameters u and v and is defined as

$$U^i_\Delta\left(\Delta^i_{obs}, \Delta^i_{req}\right) := \left(1 - \frac{1}{1 - 10 \cdot u \cdot \left(\Delta^i_{obs} - \Delta^i_{req} + \Delta^i_{tolerance}\right)}\right) \quad (6)$$
$$\cdot\left(1 + 10 \cdot v \cdot \left(\Delta^i_{obs} - 2 \cdot \Delta^i_{req} + \Delta^i_{tolerance}\right)\right)$$

with $U^i_\Delta\left(\Delta^i_{obs}, \Delta^i_{req}\right) := 0$ if $\Delta^i_{obs} \leq \Delta^i_{req} - \Delta^i_{tolerance}$. The maximum length of allocation periods is related to the parameter $\Delta^i_{tolerance}$, which is defined as

$$\Delta^i_{tolerance} = \frac{\sqrt{(10 \cdot v)^2 + 10 \cdot u \cdot 10 \cdot v} - 10 \cdot v}{10 \cdot u \cdot 10 \cdot v}, \quad u, v \in \mathbb{R}^+, \quad u, v > 0$$

and reflects the tolerated variation of the delay of resource allocations, i.e. TXOPs.

The utility U^i_Δ can be compared to a mirrored U^i_Θ function. This reflects that a high Θ^i_{obs} and a low Δ^i_{obs} are preferable for a player. Real time applications require constant allocation periods. Thus U^i_Δ is reduced for $\Delta^i_{obs} > \Delta^i_{req}$ and in addition to this it is not useful to have very short allocations, i.e. $\Delta^i_{obs} \ll \Delta^i_{req}$. The utility reaches its maximum value for $\Delta^i_{obs} = \Delta^i_{req}$ und decreases for small periods of resource allocation.

Fig. 2 shows a utility function U^i for a QoS requirement of $(\Theta^i_{req},\Delta^i_{req}) = (0.4, 0.045)$. Here the ideal case is assumed that no opponent player is present. Consequently, player *i* is observing its demand and its allocations are not delayed.

In the case of demanding the requirement the player maximizes its utility. Consequently, a utility maximizing action can be defined as $\hat{a}^i := (\hat{\Theta}^i_{dem},\hat{\Delta}^i_{dem}) = (\Theta^i_{req},\Delta^i_{req}) = (0.4, 0.045)$ in the case of an exclusive utilization of the resource.

IV. Payoff as Utility under Competition

The utility is introduced as a representative for the QoS a player observes depending on its demand. To evaluate the outcome off an SSG under competition the opponent's action has to be considered. Therefore, the payoff V^i of player *i* in stage *n* is defined as

$$V^i\left(\underline{a}\right) := \left(a^i, a^{-i}\right) \to U^i\left(a^i\right) \quad a^i \in A^i, a^{-i} \in A^{-i} \tag{7}$$

The payoff as outcome of the stage completes the SSG and highlights the dependency of player *i*'s payoff V^i on the opponent's action a^{-i}.

Fig. 3 shows the payoff V^i of player *i* in stage *n* depending on its action $a^i = (\Theta^i_{dem},\Delta^i_{dem})$. The opponent *-i* has a corresponding payoff function $V^{-i}\left(\underline{a}\right)$. By comparing Fig. 2 and Fig. 3, which are equally scaled, the mutual influence of the players' actions gets obvious: due to the competition, the players observe less than demanded. The figures lead to the best response action of a rational player: When demanding resources a player has to consider the expected action of its opponent. This is a response to the opponent's action. During the game the players do not know which action the opponent will perform. Players assume that the opponent's last stage action will be identical to the present action. The rational behavior of the opponent leads to an action, which is based on the opponent's assumption that all players act rational.

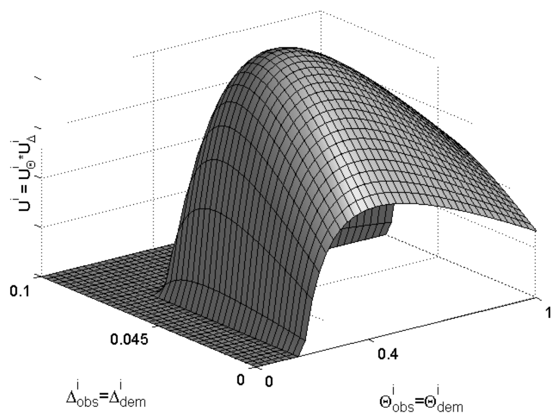

Fig. 2: The utility function U^i vs. the observation $(\Theta^i_{obs},\Delta^i_{obs})$ of player *i*, with $(\Theta^i_{req},\Delta^i_{req}) = (0.4, 0.045)$.

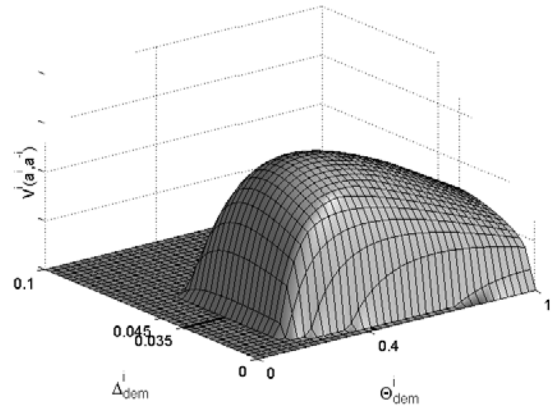

Fig. 3: The payoff function $V^i\left(\underline{a}\right)$ vs. the demand $(\Theta^i_{dem},\Delta^i_{dem})$ of player *i*, with $(\Theta^i_{req},\Delta^i_{req}) = (0.4, 0.045)$. Here, player *-i* demands $(\Theta^{-i}_{dem},\Delta^{-i}_{dem}) = (0.4, 0.02)$. Player *i* fails to receive its requirement due to the opponent's presence. Thus the competition leads to a payoff reduction. The action $(\hat{\Theta}^i_{dem},\hat{\Delta}^i_{dem}) = (0.4, 0.035)$ maximizes player *i*'s payoff and can be seen as a best response on the opponent's action.

A best response action $(\hat{\Theta}^i_{dem},\hat{\Delta}^i_{dem})$ is defined as

$$\left(\hat{\Theta}^i_{dem},\hat{\Delta}^i_{dem}\right) := max_{\Theta^i_{dem},\Delta^i_{dem}}\left[V^i\left(\Theta^i_{dem},\Delta^i_{dem},\tilde{\Theta}^{-i}_{dem},\tilde{\Delta}^{-i}_{dem}\right)\right].$$

Fig. 3 illustrates the best response of player *i* as the maximum of its payoff $V^i\left(\underline{a}\right)$ in our two-player example for the utility functions presented above.

Players use an analytic Markov model [7] together with their belief about the opponent's expected action of the actual stage to calculate their potential payoffs and thus their best response. The believe about the opponent's action is founded on an analysis of the game history H^n, i.e. the correlation of observed opponent allocations [6].

V. Equilibrium Analysis of Single Stage Game

The outcome of an SSG, namely the payoff as defined in Equation (7), depends on the observed QoS parameter which can be determined by the players through an analytic model [7]. It is interesting to analyze whether these outcomes are steady and/or payoff maximizing. Therefore, in an SSG of rational acting players the existence of a best response action, denoted $\hat{a}^i$ for player *i*, on the expected opponent's action has to be considered. In addition, the uniqueness and stability of such an action is of interest for the players' decision making process, which action to take. A commonly used solution concept for the question which action should be selected in an SSG of rational players is the NE solution concept [8], [9].

In general, a NE is a profile of strategies such that each player's strategy is a best response to the other players' strategy. Here, in the context of the SSG the players' strategy consists of a single specific action. This action leads to an observed payoff, as outcome of the SSG. NEs are consistent predictions of how the game will be played. In the sense, if all players predict that a particular NE will occur, then no player has the incentive to play differently. Thus, an NE, and only an NE, can have the property that the players can predict it, predict that their opponents predict it, and so on. The NE is a value for the game's stability. Thus it can be seen as a lower limit for the QoS that can be guaranteed in a competition scenario of rational players.

The microeconomic concept to judge outcomes of a game is the Pareto efficiency [10]: An SSG outcome is called Pareto efficient if neither player can gain a higher payoff without decreasing the payoff of at least one other player. A non-Pareto efficient situation is not a preferable outcome of a stage because a rational player could improve its payoff without changing the game and its outcome of the other players.

The bargaining domain of Fig. 4 contains a subset of all possible SSG outcomes, by means of players' payoffs $(V^i \mid V^{-i})$, corresponding to an action pair $(a^i \mid a^{-i})$. Here, the actions are from a discrete action space and the corresponding SSG outcomes, i.e. payoff pairs, are calculated with the help of an analytic model [7]. The bargaining domain supports the judgment of potential SSG outcomes.

Depending on the players' requirements $(\Theta^i_{req}, \Delta^i_{req})$ and $(\Theta^{-i}_{req}, \Delta^{-i}_{req})$, none, a unique NE or several NEs can be found. In an SSG with one NE, which is not Pareto efficient, this equilibrium can be considered as a minimum for the reachable payoffs of both players. In this way a lower but nevertheless predictable limit for the support of QoS is given. A further analysis indicates that this unique NE is reached as a steady outcome of an MSG if both players follow the "Best Response" behavior.

The single NE enables a definition of a Pareto Frontier, which marks the reachable outcomes under a "Best Response" behavior.

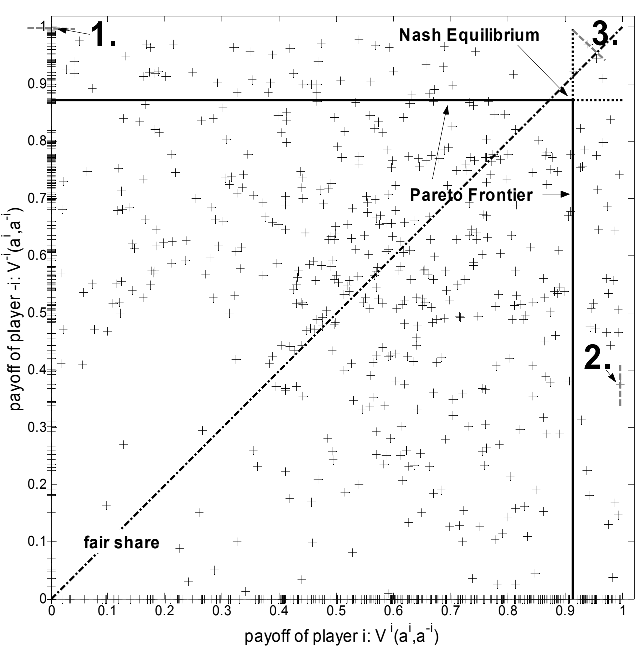

Fig. 4: Bargaining domain of the SSG. Each cross marks a payoff outcome of an SSG. The only NE in this example belongs to the actions $(\Theta^i_{dem}, \Delta^i_{dem}) = (0.5, 0.032)$ and $(\Theta^{-i}_{dem}, \Delta^{-i}_{dem}) = (0.54, 0.03)$, leading to payoffs of $(0.913 \mid 0.872)$. It characterizes the Pareto Frontier of the bargaining domain. The upper right corner is preferred by both players and contains the Pareto efficient outcome.

All outcomes outside the Pareto Frontier are characterized by Pareto domination of the NE. The illustrated SSG has three Pareto efficient outcomes: the payoff pairs with a maximum payoff for each player located for player *-i* in the upper left area of the bargaining domain (see 1) and for player *i* in the lower right area (see 2). The third Pareto efficient outcome is located in the upper right area (see 3) with the longest distance to the origin of the bargaining domain. There, both players gain, contrary to the other two Pareto efficient outcomes, a higher payoff than in the NE. As a result, both players can improve their payoffs through interaction, compared to payoffs in the NE. This interaction is referred to as cooperation to the benefit of all players.

In the remaining part of this section the behavior of a player is considered. Therefore, all to a player available actions, i.e. all combinations of demanded QoS parameters Θ_{dem} and Δ_{dem}, are summed up within an action portfolio of Fig. 5. They are judged under consideration of behavior and their consequences on the opponents. In each corner an exemplary allocation scheme is depicted to illustrate the dependency of the TXOP allocations on the demanded QoS parameters.

A player *i* showing a behavior of "Best Response" selects the action corresponding to the highest expected payoff in the SSG. A "Best Response" behavior leads to an action which implies an increase of the demand compared to the requirement, i.e. $\Theta^i_{dem} > \Theta^i_{req}$ and $\Delta^i_{dem} < \Delta^i_{req}$.

A player *i* showing the behavior of "Cooperation" attempts to gain higher payoffs than in a game of "Best Response" acting players. The "Cooperation" behavior allows the opponent player *-i* to meet better its requirement without the knowledge about the opponent's requirement. In the case of a same behaving opponent, all players gain from this cooperation. A cooperating player selects $\Theta^i_{dem} = \Theta^i_{req}$ and $\Delta^i_{dem} < \Delta^i_{req}$.

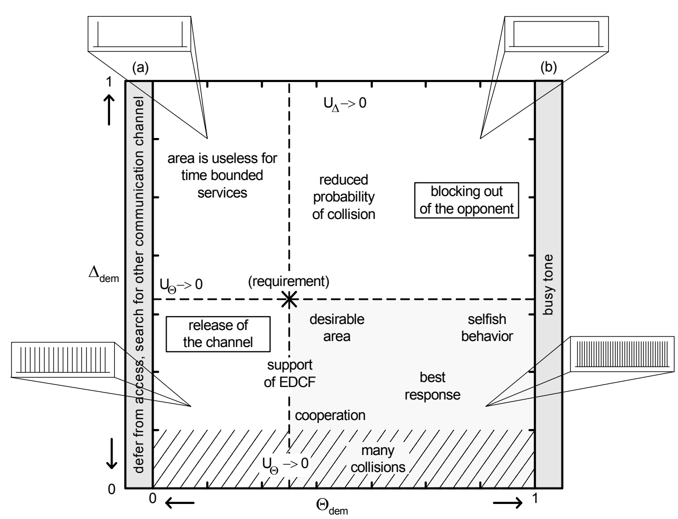

Fig. 5: Portfolio of available actions, the corresponding utilities and the resulting consequences on the opponents. Exemplary allocation schemes are depicted to illustrate the dependency of the allocations on the demanded QoS parameters. The case of (a) can be compared to a leaving of the MSG and (b) occupying all resources for all time, are not part of the game structure.

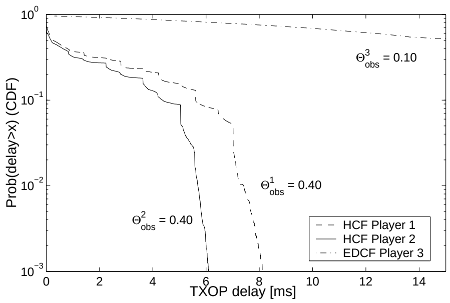

Fig. 6: CDF of TXOP (MSDU delivery) delays when two QBSSs operate in parallel. QoS guarantee is not feasible.

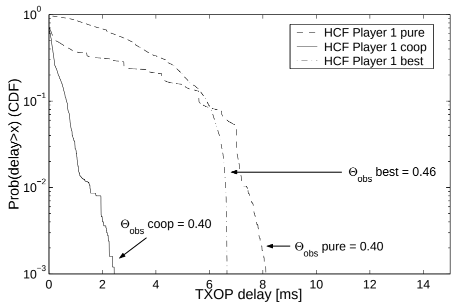

Fig. 7: CDF of TXOP (MSDU delivery) delays of player *1* for three different behaviors: (1) static 802.11e, (2) game wide "Best Response" behavior and (3) game wide "Cooperation" behavior.

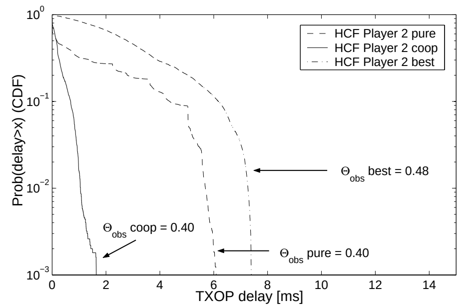

Fig. 8: CDF of TXOP (MSDU delivery) delays of player *2* for three different behaviors.

VI. MULTI STAGE GAMES

The analysis of SSGs has shown that all players can benefit from a dynamic interaction. Therefore, in this section the focus is on MSGs that are formed by repeated SSGs. Within such an MSG the above-introduced behaviors are evaluated.

We define a scenario of two completely overlapping QBSSs. Two QBSSs, represented through player *1* and *2* are trying to support QoS. Player *1* carries QoS with the parameters of $\Theta^1_{req} = 0.4$ and $\Delta^1_{req} = 0.075$, while player *2* has QoS requirements of $\Theta^2_{req} = 0.4$ and $\Delta^2_{req} = 0.085$. A third player represents the EDCF traffic of both QBSSs with an offered load of $5\ Mbit/s$ and an EDCF TXOPlimit of *1.1 ms*.

In such a competition scenario a satisfying guarantee of QoS cannot be given. The competitive access of player *1* and *2* to the shared resource leads to severe delays of their TXOPs, i.e. MSDU delivery delays, as depicted in Fig. 6.

Through the application of behaviors the coexistence problem may be solved. Fig. 7 and Fig. 8 depict a comparison of three games with different behaviors for player *1* and *2*: pure 802.11e vs. pure 802.11e, Cooperation vs. Cooperation and Best Response vs. Best Response. The scenario of pure 802.11e behaviors reflects the in Fig. 6 introduced coexistence problem of overlapping QBSSs. If both players follow a "Best Response" behavior they have a limited ability to guarantee QoS. They both try to block each other out of the shared resource and interfere. Here contrary to many other scenarios of different behaviors, a "Best Response" behavior is inadequate to guarantee a satisfying QoS. In a scenario of a game wide "Cooperation" behavior all players can profit: They all are able to guarantee a satisfying QoS.

VII. CONCLUSION

The "Best Response" behavior, based on a player's analysis of the SSG, is adequate guarantee QoS independently from the opponent's. The level is limited, but nevertheless predictable. A game wide cooperation is to the benefit of all players and thus preferable to reach. Nevertheless, exists a temptation to gain a short term higher payoff in deviating from this cooperation.

The introduced stage-based concept is a promising approach to solve the coexistence problem of overlapping QBSSs. A refinement of the outlined behavior under consideration of a further interaction leads to a definition of strategies and may improve the ability of the players to guarantee QoS on a satisfying level.

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