Theoretical Analysis of Saturation Throughput in MU-DCF

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***Abstract*—In this paper, the calculation of saturation throughput for previously proposed Multi-User - Distributed Coordination Function (MU-DCF) is presented. MU-DCF is an IEEE 802.11 based protocol that supports multi-user (MU)-Multiple Input-Multiple Output (MIMO) transmissions.**

The analysis is for two extreme case scenarios: Access Point (AP) downlink in a hotspot scenario, and fully interconnected network. Special attention is payed on the statistical properties of traffic sources, since in MU-DCF networks they have strong impact on performance. In addition, the difference in performance between MU and single-user (SU) transmission strategies is evaluated, pointing out to the tradeoff between delay and throughput.

I. Introduction

Being a de-facto standard for Wireless Local Area Networks (WLANs), IEEE 802.11 protocols [1] are a hot research topic for both further enhancements, such as currently IEEE 802.11n [2], as well as performance analysis of the current standards. A model based on two-dimensional Markov chains has been proposed by Bianchi in [3] for the saturation throughput analysis of IEEE 802.11 WLANs. The model has been further extended for delay evaluation in [4]. Moreover, a full statistical characterization of service time based on Z-transform has been derived in [5].

MU-DCF [6] is a Medium Access Control (MAC) protocol with MIMO support based on IEEE 802.11 DCF. In contrast to IEEE 802.11n protocol, it supports MU-MIMO transmissions. Besides giving an additional degree of freedom for packet scheduling, important benefits of MU transmissions are potential to reduce the number of time the channel has to be accessed, and improved fairness and delay properties [7].

In this paper, Bianchi model has been modified for the analysis of saturation throughput of MU-DCF. Due to the multiple possible realizations when transmitting a MIMO frame, depending on the number of its distinct receivers, the saturation throughput directly depends on the packet scheduler; in this work it has been assumed that the packets are scheduled in First In - First Out (FIFO) order, and therefore the performance depends on the statistical properties of the individual traffic sources. The performance analysis has been done for several traffic types, varying the coefficient of variation (CoV) of distribution of packet interarrival times at stations.

The paper has the following structure: in Section II, descriptions of MU-DCF and Single-User - Distributed Coordination Function (SU-DCF) are given. In Section III, the system model is described. The throughput study is presented in Section IV, for the hotspot scenario in Subsection IV-A, and fully interconnected network in Subsection IV-B. Discussion of results, together with conclusion is given in Section V.

II. SU-DCF and MU-DCF Description

Both SU-DCF and MU-DCF enhance the IEEE 802.11 Distributed Coordination Function (DCF) [1] with MIMO capability. SU-DCF is a special case of MU-DCF; therefore MU-DCF is described in detail, followed by the restrictions specific to SU-DCF.

MU-DCF [6] provides the protocol support for MU-MIMO transmissions. It facilitates channel access with a four-way handshake procedure with multiple users prior to data transmission. Similarly as in IEEE 802.11 DCF, medium access in MU-DCF is based on Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA), with a random backoff procedure.

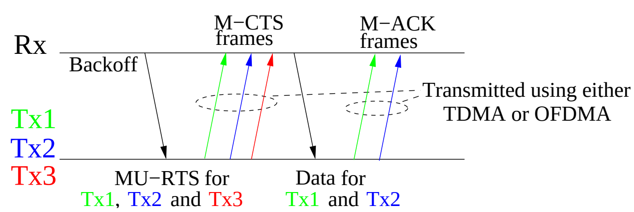


Fig. 1. SU- and MU-MIMO frames

The IEEE 802.11 standard [1] includes an optional Request-to-Send (RTS) / Clear-to-Send (CTS) handshake prior to the transmission to alleviate the hidden station problem and reserve the medium for data transmissions. Similarly, MU-DCF uses extended forms of RTS and CTS: MU-RTS and MIMO-CTS (M-CTS). They are used not only to solve the hidden station problem, but also to exchange information about multiple antennas. MU-RTS / M-CTS handshake is optional in MU-DCF as well. MIMO-ACK (M-ACK), the extended Acknowledgment (ACK) frame is used to acknowledge the data frames (each frame is acknowledged separately).

Setting the Network Allocation Vector (NAV) timer is done as in IEEE 802.11, as well as the usage of interframe spaces. All the control frames are transmitted using an antenna scheme that is supported by all the stations, independently of their hardware capabilities, including the stations with only one antenna.

It is worth noting that the stations that are addressed in the MU-RTS do not necessarily have to be receivers of data frames within a MIMO frame (e.g. if the channel is in bad state), as illustrated in Figure 1. This decision is made by the transmitter that does the final scheduling after receiving the M-CTS frames. Also, the opposite applies: not addressing a station in the MU-RTS frame does not mean that a data frame will not be transmitted to it in the next MIMO frame.

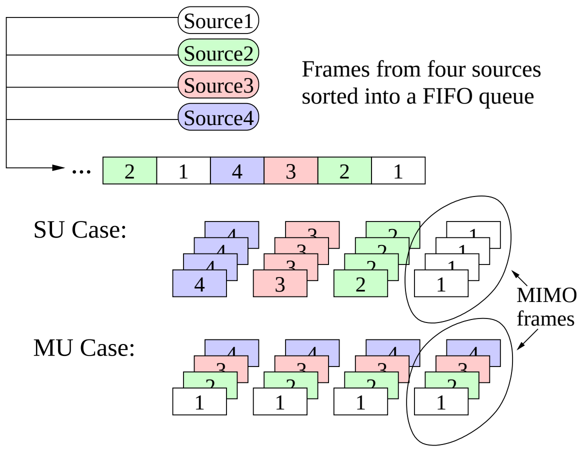


Fig. 2. SU- and MU-MIMO frames

In order to reduce the signaling overhead, multiple M-CTS and M-ACK frames can be transmitted using Orthogonal Frequency Division Multiple Access (OFDMA) instead of Time Division Multiple Access (TDMA). The benefits of this approach have been demonstrated in [6].

The essential features of MU-DCF are:

- MU-MIMO transmission in IEEE 802.11 fashion.
- Support for fast link adaptation.
- MU-DCF is scalable, and provides backward compatibility, coexistence and interoperability to stations with different number of antennas (including single-antenna stations implementing conventional IEEE 802.11 DCF).

SU-DCF is the special case of MU-DCF, when it operates under the restriction that MIMO frames consist only of data frames addressed to the same destination, as illustrated in Figure 2. In contrast to MU-RTS frames, SU-RTS frames address only *one* station. The other control frames of SU-DCF are the same as in MU-DCF.

III. System Model

In this Section the saturation throughput is analyzed, depending on the number of stations for the following protocols:

- standard IEEE 802.11 DCF (SISO),
- SU-DCF (SU-MIMO), and
- MU-DCF (MU-MIMO), both TDMA and OFDMA based versions of the protocol.

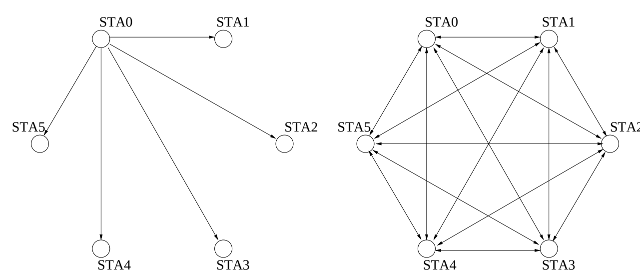
Only the results for basic medium access are presented, since it is used predominantly in practice; applying four-way handshake will solve the hidden station problem, and mitigate the throughput degradation owing to collisions.

The influence of the traffic characteristic is considered by examining network performance for fixed, exponentially and hyper-exponentially distributed packet interarrival times at stations, with varying CoV.

The following two scenarios are studied, both with the number of stations n being a parameter.

1) Scenario I is presented in Figure 3(a) for $n = 6$. Only one station in the network, `STA0`, is transmitting, and it has $n-1 = 5$ unidirectional (downlink) connections with all the other stations in the network. This scenario is used to study the satiration throughput of different MIMO transmission strategies, without taking into account the impact of the medium access procedure.
2) Scenario II, presented in Figure 3(b) is a fully interconnected network: each station establishes connections with all the other stations in the network. In the throughput study, the differing effects of the medium access procedure on the protocols can be seen.

Throughout the analysis, it is assumed that the channel is error free, and that erroneous frame receptions happen only as a consequence of collision of multiple frames. Thus, erroneous receptions happen only in Scenario II. Hidden and exposed station problem do not occur. It is assumed that the stations' data queues are at all times nonempty, more specific, in the data queue there is at least one frame in DCF, at least four frames for the receiver whose frame is the head-of-line in SU-DCF, and in MU-DCF at least four frames, independently of their destination. These assumptions are similar to those made in Bianchi model [3], [4], including the discrete integer time scale, and lead to constant and independent conditional collision probability p, seen by a data frame being transmitted on the channel. The relevant parameters are listed in Table I.



(a) One station transmitting (b) Fully interconnected network

Fig. 3. Analyzed scenarios

IV. Throughput Study

A. Scenario I: AP Downlink

In Figure 4, the saturation throughput at `STA0` in Scenario I is presented vs. the number of stations. In all the cases, it is assumed that all the connections have the same constant data frame size (1024 byte), the same intensity of the offered load, as well as the same distribution of packet interarrival times at stations.

In DCF system, the saturation throughput $S_{\text{AP}}^{\text{DCF}}$ can be easily calculated as the ratio of the average data frame length and the duration of the transmission window T_s^{DCF} increased by

TABLE I

SYSTEM PARAMETERS

Parameter	Value
Channel Bandwidth	20 MHz @ 5.2 GHz
Number of Subcarriers	48 Data + 4 Pilot
Slot duration (σ)	9 μs
SIFS/DIFS/EIFS	16 μs / 34 μs / 94 μs
PHY Mode	64 QAM $^3/_4$ (54 $^{Mbit}/_s$)
CW_{min}	15
Maximum backoff stage	6
Data frame length	1024 byte
Number of Tx / Rx antennas	4 / 4 (in MIMO systems)

the average duration of the backoff countdown. The notation from [3] has been adopted, therefore s in subscript stands for successful transmission (and in the next sections c will denote a collision):

$$S_{\text{AP}}^{\text{DCF}} = \frac{\text{E}[P]}{\frac{\text{CW}_{\text{min}}}{2}\sigma + T_s^{\text{DCF}}}$$
$$T_s^{\text{DCF}} = T_{\text{DIFS}} + \text{E}\left[T_{\text{data}}\right] + T_{\text{SIFS}} + T_{\text{ACK}} \quad (1)$$

In the previous equations $\text{E}[\cdot]$ stands for the expectation value, P for the data frame length (payload size), CW_{min} is the starting - minimum Contention Window (CW) size, and σ is the slot duration. For the Physical layer (PHY) parameters and data frame length given in Table I, the saturation throughput takes the following value:

$$S_{\text{AP}}^{\text{DCF}} = 25.48\ ^{Mbit}/_s$$

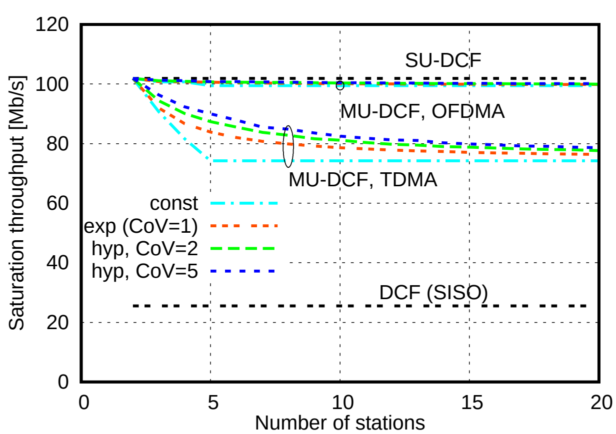


Fig. 4. Saturation throughput in Scenario I

In SU-DCF system, four data streams are transmitted simultaneously, and the duration of the transmission window corresponds to time needed for the longest frame out of four to be transmitted:

$$S_{\text{AP}}^{\text{SU}} = \frac{4\cdot \text{E}[P]}{\frac{\text{CW}_{\text{min}}}{2}\sigma + T_s^{\text{SU}}}$$
$$T_s^{\text{SU}} = T_{\text{DIFS}} + \text{E}\left[max_4\{T_{\text{data}}\}\right] + T_{\text{SIFS}} + T_{\text{ACK}}$$

In the analyzed example, the data frame length is constant, therefore it applies:

$$T_s^{\text{SU}} = T_s^{\text{DCF}} \Rightarrow S_{\text{AP}}^{\text{SU}} = 4\cdot S_{\text{AP}}^{\text{DCF}} = 101.92\ ^{Mbit}/_s \quad (2)$$

For the two cases analyzed up to now, the throughput neither depends on the number of stations, nor on the traffic type (which determines the order of frames in the data queue), since the transmission window is fixed, and therefore has constant duration; this will not be the case with MU-DCF.

Duration of a transmission window in MU-DCF depends on how many distinct receivers are addressed in the MIMO frame. If we assume that the MIMO frames are generated from the data queue obeying the FIFO order, the number of distinct receivers depends on the number of stations in the network, as well as on the distribution of packet interarrival times for each connection. In Figure 5 number of distinct receivers vs. number of connections is plotted for different traffic load types. In the following, the calculation/analysis for the individual cases is given.

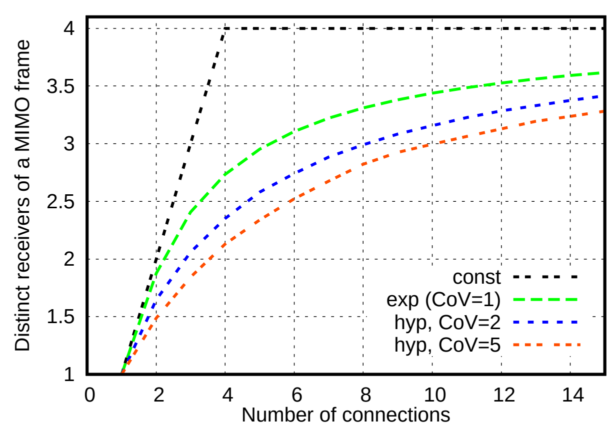


Fig. 5. Number of distinct receivers of a MIMO frame vs. number of connections for different traffic types in MU-DCF

1) *Constant bitrate*
Under constant bitrate, the number of distinct users of each MIMO frame d is $min(n-1, 4)$. Since the frames from different sources always appear in the same order in the data queue, as soon as the number of receivers becomes greater or equal four, the AP will operate in pure MU mode.
2) *Poisson traffic*
In the following, the calculation of the probability mass function $P_d(i)$ of the random variable d is presented, in case that all the sources have exponential distribution of the packet interarrival times at stations. If the number of the stations in the network is n, and therefore the

number of connection is $n-1$, the probability that the head of the line is addressed to a specific receiver is $\frac{1}{n-1}$, owing to the memoryless property of the exponential distribution. To calculate the probability mass function, it has to be taken into account that the permutations of the four frames differ only if the particular receivers differ, independently of the frame sequence number (denominators of the "small" fractions in numerators incorporate this). According to the definition of the problem, the probability mass function is 0 for all the other values but 1, 2, 3 and 4:

$$\begin{aligned}
P_d(1) &= \binom{n-1}{1} \cdot \frac{\frac{4!}{4!}}{(n-1)^4} \\
P_d(2) &= \binom{n-1}{2} \cdot \frac{\frac{4!}{1!\cdot 3!} + \frac{4!}{2!\cdot 2!} + \frac{4!}{3!\cdot 1!}}{(n-1)^4} \\
P_d(3) &= \binom{n-1}{3} \cdot \frac{\frac{4!}{1!\cdot 1!\cdot 2!} + \frac{4!}{1!\cdot 2!\cdot 1!} + \frac{4!}{2!\cdot 1!\cdot 1!}}{(n-1)^4} \\
P_d(4) &= \binom{n-1}{4} \cdot \frac{\frac{4!}{1!\cdot 1!\cdot 1!\cdot 1!}}{(n-1)^4}
\end{aligned}$$

The equations apply for each $n > 1$, since $\binom{n}{k} = 0$, when $k < n$. Using these results, the average number of distinct receivers can be calculated:

$$\mathrm{E}[d] = \sum_{i=1}^{4} i \cdot P_d(i)$$

The average number of distinct receivers $\mathrm{E}[d]$ when assuming Poisson load has been plotted in Figure 5. Although the traffic is bursty, d increases with the number of stations, since the probability that their bursts happen simultaneously, and therefore get mixed in the data queue grows. Similarly to the calculation of $\mathrm{E}[d]$, calculation of saturation throughput is done:

$$\begin{aligned}
S_{\text{AP}}^{\text{MU}} &= \frac{4 \cdot \mathrm{E}[P]}{\frac{\text{CW}_{\text{min}}}{2}\sigma + \mathrm{E}\left[T_s^{\text{MU}}\right]} \\
\mathrm{E}\left[T_s^{\text{MU}}\right] &= \sum_{i=1}^{4} P_d(i) \cdot T_{s,i}^{\text{MU}}
\end{aligned}$$

where $T_{s,i}^{\text{MU}}$ is the duration of the transmission window when $d = i$. Its value differs for TDMA and OFDMA based signaling:

$$\begin{aligned}
T_{s,i}^{\text{MU}_{\text{TDMA}}} &= T_{\text{DIFS}} + \mathrm{E}\left[max_4\{T_{\text{data}}\}\right] + \\
&\quad i \cdot (T_{\text{SIFS}} + T_{\text{M-ACK}}) \\
T_{s,i}^{\text{MU}_{\text{OFDMA}}} &= T_{\text{DIFS}} + \mathrm{E}\left[max_4\{T_{\text{data}}\}\right] + \\
&\quad T_{\text{SIFS}} + T_{\text{M-ACK}_i}
\end{aligned}$$

where $T_{\text{M-ACK}_i}$ is the time needed for i M-ACK frames to be transmitted over the channel in parallel using OFDMA.

3) *Traffic sources with hyper-exponentially distributed packet interarrival times at stations*
In order to evaluate the performance of the protocol when the traffic is even more bursty, the traffic sources with hyper-exponentially distributed packet interarrival times at stations have been assumed. Since hyper-exponential distribution is different from the exponential distribution in that it has a memory, the results for this case have been obtained by simulating the packet arrival times of individual sources, and measuring the probability of different transmission window realizations.

It can be seen from Figure 4 that as the number of stations in the network grows, MU systems' saturation throughput decreases. The opposite happens as the traffic gets more bursty. SU system performance gives the upper bound (Equ. (2)), and the MU-DCF under constant bitrate gives the lower bound for the saturation throughput:

$$\begin{aligned}
S_{\text{AP, min}}^{\text{MU}_{\text{TDMA}}}(n-1)|n-1 \geq 4 &= 74.22\, ^{Mbit}/_{s} \\
S_{\text{AP, min}}^{\text{MU}_{\text{OFDMA}}}(n-1)|n-1 \geq 4 &= 99.45\, ^{Mbit}/_{s}
\end{aligned}$$

It is interesting to compare these values with the saturation throughput of the SU-DCF from Equ. (2) and note that using TDMA significantly deteriorates the saturation throughput, in contrast to OFDMA based signaling.

The saturation throughput of the observed MIMO systems are mutually equal only when there are only two stations in the network, and this value coresponds to the SU saturation throughput.

In the previous analysis, specific traffic types have been assumed. However, the derived formulas apply in general; for arbitrary load sources only the probability mass function P_d will change.

B. Scenario II: Fully Interconnected Mesh Network

For the analysis of the saturation throughput in a fully interconnected network, Bianchi model for IEEE 802.11 DCF networks [3], [4] has been applied, and modified for the calculation of the saturation throughput of the MIMO systems. The results are plotted in Figure 6.

According to Bianchi model, the saturation throughput $S_{\text{mesh}}^{\text{DCF}}$ is:

$$S_{\text{mesh}}^{\text{DCF}} = \frac{P_s P_{tr} E\left[P\right]'}{(1-P_{tr})\,\sigma + P_{tr} P_s T_s'^{\text{DCF}} + P_{tr}\,(1-P_s)\,T_c'^{\text{DCF}}}$$

where $P_{tr} = 1 - (1-\tau)^n$ is the probability that there is at least one ongoing transmission in the considered virtual slot with $\tau = 1 - (1-p)^{\frac{1}{n-1}}$ being the probability that a station transmits, and p conditional collision probability. $P_s = \frac{n\tau(1-\tau)^{n-1}}{P_{tr}}$ is the probability that the ongoing transmission finishes successfully. $E[P]'$ is the expected number of bytes transmitted in data frames within a virtual slot:

$$\mathrm{E}\left[P\right]' = \mathrm{E}\left[P\right] + \sum_{i=1}^{\infty} B_0^k \mathrm{E}\left[P\right] = \frac{\mathrm{E}\left[P\right]}{1-B_0}$$

where $B_0 = {}^{1}/_{\text{CW}_{\text{min}}}$ is the probability that after a successfull transmission, a station draws zero for the new value of the backoff counter. $T_s'^{\text{DCF}}$ and $T_c'^{\text{DCF}}$ are the average durations of successfull transmission slot and collision slot:

$$\begin{aligned} T_s'^{\text{DCF}} &= T_s^{\text{DCF}} + \sum_{i=1}^{\infty} B_0^k T_s^{\text{DCF}} + \sigma = \frac{T_s^{\text{DCF}}}{1-B_0} + \sigma \quad (3) \\ T_c' &= T_c + \sigma \\ T_c &= \mathrm{E}\left[max_{16}\{T_{\text{data}}\}\right] + T_{\text{EIFS}} \end{aligned}$$

using T_s^{DCF} given in Equ. (1).

In order to determine the saturation throughput for the MIMO systems at hand, in Equ. (3) instead of T_s^{DCF}, T_s^{SU}, $\mathrm{E}\left[T_s^{\text{MU}_{\text{TDMA}}}\right]$ and $\mathrm{E}\left[T_s^{\text{MU}_{\text{OFDMA}}}\right]$ are applied, for MU case depending on the traffic type. The values corresponding to constant bitrate and Poisson load are calculated, and plotted in Figure 6, together with the results for the hyper-exponentially distributed packet interarrival times at stations, obtained by simulations. All the systems suffer from the increased number of collisions as the network size grows. The higher the original saturation throughput when the number of stations is small, the more the system suffers from the collisions resulting from multiple transmissions. Whilst with 5 stations in the network, the difference between the saturation throughput of SU-DCF and the lower bound for the TDMA based MU-DCF is 30 $^{Mbit}/_{s}$, the difference reduces with 15 stations to about 20 $^{Mbit}/_{s}$.

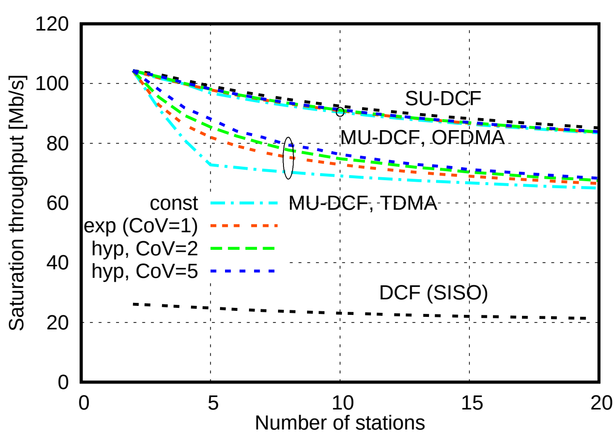


Fig. 6. Saturation throughput in Scenario II

V. Conclusion

In this work the saturation throughput analysis of MU-DCF is presented. The difference of SU and MU transmission strategies is quantitatively evaluated for the OFDM PHY (IEEE 802.11a standard [1]) with highest bitrate. For lower valued PHY modes the bitrate decreases and the difference in performance of OFDMA and TDMA based MU-DCF becomes less significant. E.g., for the most robust OFDMA PHY mode, BPSK $1/2$, the saturation throughput lower bounds are 18.31 $^{Mbit}/_{s}$ in case of TDMA and 19.49 $^{Mbit}/_{s}$ in case of OFDMA. Moreover, the difference in performance compared to SU-DCF (20.36 $^{Mbit}/_{s}$) is also not large. However, if the channel conditions are so bad that such robust PHY mode has to be used, then it might be a better option to apply some other MIMO techniques such as different diversity schemes or beamforming, than using spatial multiplexing.

A similar effect can be observed when the data frame size increases, since in that case the relative overhead is decreased. However, many applications, particularly interactive ones, or multimedia streaming, have very small payload, and aggregation is not always applicable, due to the packet lifetime restrictions.

The results of the analysis give evidence that in the system in overload conditions higher throughput can be achieved using SU than using MU transmission strategy. However, in the complete performance evaluation of a system, other metrics have to be examined too. In [7], a comparison of fairness and delay properties has been given. Due to the less restrictive scheduling policy, MU systems are significantly better in providing short-term fairness, and this translates also into better delay characteristic: with MU approach, the system benefits from multiple connections, even when the offered load is increasing, which is not the case with SU approach; for the constant offered load, the delay characteristic of the MU system does not degrade if the number of connections grows, whereas the delay in the SU system grows linearly. The analysis in [7] has been done for the system that is not fully loaded, but particularly in the saturated system, the delay has to be treated separately, so that the packet lifetime is not exceeded, which would lead to a frame being discarded already at the transmitter.

Taking into account these properties, a "smart" scheduler will build MIMO frames making a proper tradeoff between the throughput on the one hand, and delay and fairness on the other one, regarding the Quality of Service (QoS) requirements of the individual connections, as well as the current state of the network.

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