Cell Spectral Efficiency Analysis of Relay Enhanced and Picocell Enhanced LTE Systems

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***Abstract*—A novel analytical framework is presented to evaluate resource consumption, error ratio, and spectral efficiency on PHY, MAC and RLC layer in relay enhanced 3GPP LTE systems. Thereby, various aspects of the various LTE protocol layers are modeled: On PHY layer, association of user terminal to a radio access point, probabilistic radio channel states, adaptive modulation and coding, overhead in LTE radio frame, turbo coded M-QAM block error rate, and Chase combining are taken into account. On MAC layer, hybrid-ARQ protocol, resource assignment failure, and feedback misdetection are accounted for. On RLC layer, segmentation of a RLC SDU into several RLC PDUs, selective repeat - ARQ protocol, feedback loss, and timer for upcoming feedback to be waited for are considered. Aspect of two hop transmission is modeled and analyzed for decode and forward relaying. The framework is useful to evaluate various performance parameters for relay enhanced cells and for picocell enhanced donor cells as well: Resource consumption, error ratio and spectral efficiency are calculated as CDFs for any location in a multi-cell scenario, as location specific expected values, as CDFs for a cell, and as related expected values and percentiles.**

I. Introduction

Spectral efficiency, namely the normalized user throughput capacity, is defined as the maximum user throughput divided by the channel bandwidth [3]. It is the number of correctly received bits normalized by the consumed resource in time and in bandwidth. Thus, spectral efficiency is strongly related to resource consumption and packet error ratio.

The 3GPP LTE system model is illustrated in Fig. 1 for downlink transmission case. On physical (PHY) layer, adaptive modulation and coding (AMC) is adopted. If radio condition is not good enough, a lower rate modulation and coding scheme (MCS) is used to reduce block error rate (BLER) at the cost of more radio resource consumption. On medium access control (MAC) and radio link control (RLC) layers, the hybrid automatic repeat request (H-ARQ) protocol and the selective repeat ARQ (SR-ARQ) protocol are applied, respectively. Thereby, errors left over from previous transmission of the same data are resolved by retransmission to reduce residual error at a cost of extra resource consumption.

Relaying technique introduced in 3GPP LTE-Advanced is also shown in Fig. 1 for downlink case. A decode and forward relay node (RN) appears as a user terminal (UT) from the donor base station's (BS) point of view, and behaves like a BS from perspective of the UTs under its control. The RN mainly forwards IP packet on the network layer, where it relies on a

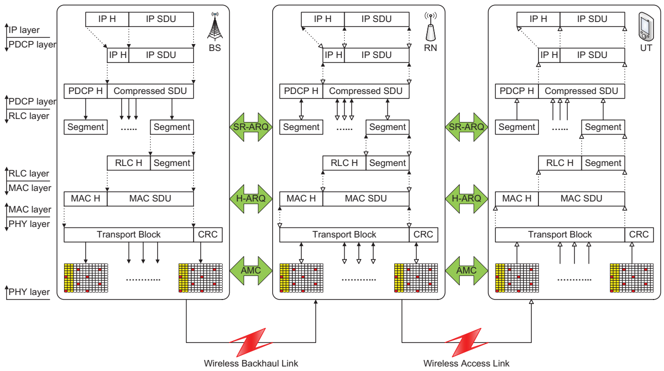


Fig. 1. Relay enhanced LTE system model

wireless backhaul link to receive user data from the BS and leans on wireless access links to transmit data to the UTs [4].

If backhaul links are operated at significantly high signal to interference plus noise ratio (SINR), which, e.g., results from line of sight (LoS) radio propagation condition between serving BS and RN and non LoS (NLoS) between interfering BSs and RN, relaying technique offers good potential to capacity improvement [5] in addition to coverage extension and shadowing reduction at relatively low cost. The spectral efficiency on two hop link can be greater than that on one hop link, so long as the total amount of resources needed on both backhaul link and access link for two hop case is less than the resources required on access link for one hop case, and the final remaining packet error rate left over from two hop transmission is less than the residual error ratio left from one hop transmission.

In this work, resource consumption, error ratio and spectral efficiency are analytically evaluated for downlink of relay enhanced LTE on system level. In particular, AMC, H-ARQ protocol, SR-ARQ protocol as well as decode and forward relaying technique are analyzed in detail using signal flow graphs (SFGs) [6].

II. Related Work

The capacity model [7] takes into account overhead caused by retransmissions resulting from packet error rate (PER), under the assumption of selective repeat automatic repeat request (SR-ARQ). PER on data link (DL) layer is calculated from bit error rate (BER) on physical (PHY) layer. For each modulation and coding scheme (MCS) a mapping between signal to interference plus noise ratio (SINR) and BER is determined from link level simulations. However, hybrid ARQ (H-ARQ) protocol with Chase combining (CC) and SR-ARQ protocol are not modeled in detail.

This model also accounts for relaying technique. The capacity of two hop link from base station (BS) over relay node (RN) to user terminal (UT) is derived from adding the reciprocal of capacity on backhaul link between BS and RN to that on access link between RN and UT, and then inversing the summation to its reciprocal. However, relaying technique is not modeled precisely following the nature of decode and forward relay.

The capacity model [8] extends the work presented in [7] to include distance dependent line of sight (LoS) and non LoS (NLoS) channel probabilities, where radio propagation condition for both signal and interference can be either LoS or NLoS. Mean SINR at a position in the cell is computed by summing over the set of all channel permutations and weighting individual SINR for each possible radio channel state by its permutation's probability. The appropriate MCS is derived from mean SINR. However, adaptive modulation and coding (AMC) is not modeled exactly for each UT position.

The analytical model to evaluate capacity on system level presented in this paper is the first one to include, among others, all aspects mentioned to be missing in the models referenced. To the best of our knowledge this is the first publication for mathematical calculation of both the spectral efficiency map of a multi-cell scenario, and cell spectral efficiency (CSE) and cell edge user spectral efficiency (CE-USE) of a cell in a relay enhanced LTE system.

III. Analytical Framework

A non-conventional framework, which consists of analytical models for physical (PHY), medium access control (MAC) and radio link control (RLC) layers as well as for two hop transmission as illustrated in Fig. 2, is applied to analyze resource consumption, error ratio and spectral efficiency on the individual layers over backhaul, access and two hop links for an relay enhanced LTE system.

Thereby, various aspects of the different LTE system layers are modeled and analyzed. On PHY layer, probabilistic radio channel states [9] in relay enhanced scenario, association of user terminals (UTs) to radio access points (RAPs), turbo coded M-QAM block error rate (BLER), adaptive modulation and coding (AMC), Chase combining (CC) and overhead in LTE radio frame for relay enhanced system [10] [11] are taken into account. On MAC layer, hybrid automatic repeat request (H-ARQ) protocol [12], resource assignment failure and feedback misdetection are accounted for. On RLC layer, segmentation of a RLC service data unit (SDU) into several RLC protocol data units (PDUs), selective repeat ARQ (SR-ARQ) protocol, feedback loss and timer for upcoming feedback to be waited for are considered. Aspect of two hop

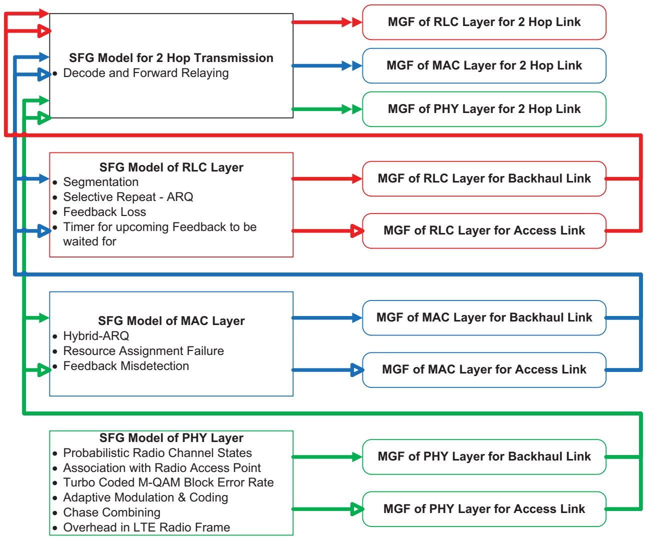


Fig. 2. Analytical framework

transmission is also modeled and analyzed, taking decode and forward relaying into consideration.

For any location in a cell the signal flow graph (SFG) model provides rigorous and detailed analysis of the transmission process of an SDU on a given layer. A layer specific moment generating function (MGF) specifies the probability distribution of radio resource consumption for SDU transmission. A MGF comprises a success part corresponding to a correct transmission and a failure part representing an erroneous transmission. The model of a given layer corresponds to two MGFs characterizing transmission on this layer over backhaul link and access link, respectively. Based on the MGF of a lower layer for a certain link, the SFG model of the next higher layer is used to analyze the MGF of this higher layer for the respective link. The SFG model for two hop transmission corresponds to three MGFs characterizing transmission over two hop link on PHY, MAC and RLC layers, respectively. On the basis of both MGFs of a certain layer, namely one for backhaul link and the other for access link, the SFG model for two hops analyzes the MGF of this layer for two hop link.

The analytical framework is capable to evaluate various types of LTE systems. An evaluation scenario without relays corresponds to a system with base station (BS) only. If a scenario is enhanced with relays, however, backhaul links are not taken into consideration, the evaluation corresponds to a relay enhanced system with wired backhaul. Please note that this system type represents the heterogeneous cell approach currently discussed for future 5G systems with picocells operated on resources of a respective donor cell. If a scenario is enhanced with relays and backhaul links are accounted for, the evaluation corresponds to a relay enhanced system with wireless backhaul. This system type is defined for current 4G standards in LTE-Advanced systems [4] [11].

Therewith, the above mentioned performance parameters are evaluated in a variety of forms. For an arbitrary small area element in a cell the probability distribution of a selected performance parameter on a certain layer is evaluated from the MGF related to this layer. Considering that UTs are assumed randomly and uniformly distributed over the whole service area, the probability distribution of some parameter is aggregated for a cell from the probability distributions valid for each area element contained in the cell. From the probability distribution, its expected value and percentiles of a certain cell related performance parameter is derived straightforwardly. The probability distribution of spectral efficiency for a cell is of particular interest, because its expected value and 5^{th} percentile are of interest to real world systems as cell spectral efficiency (CSE) and cell edge user spectral efficiency (CE-USE), respectively [3]. Scenario maps representing a certain performance parameter are plotted, where an expected parameter value is calculated from its probability distribution for each potential UT position in the scenario.

SFG models and MGFs of PHY, MAC and RLC layers are available from [1]. Therefore, only the SFG model for MAC layer and its corresponding MGF are repeated and the others are omitted here. Analytical formulas to evaluate probability distributions of resource consumption, error ratio and spectral efficiency based on MGF published in [2] are given here again. Complementary to [1] and [2], probabilistic radio channel states in relay enhanced scenario, overhead in LTE radio frame for relay enhanced system, SFG model for two hop transmission and its corresponding MGF are presented in this paper.

A. *Signal Flow Graph Model of MAC Layer*

Referring to the H-ARQ protocol [12], the SFG for MAC layer is shown in Fig. 3. The SFG, which specifies the transmission of MAC SDU within maximal N times transmission of a transport block on PHY layer, is recursively defined based on the SFG, which describes the MAC transmission for maximal $N-1$ times PHY transmissions.

Firstly, the transmission on MAC layer distinguishes between successful and unsuccessful resource assignment, where p_{ASSIGN_ERR} stands for the probability of resource assignment failure. Secondly, each branch differentiates between two groups, which represent correct and erroneous transmission on PHY layer, respectively. Thirdly, ACK represents the feedback for correct transmission, NAK for erroneous transmission and DTX for resource assignment failure. However, the feedback for any case can be detected or misdetected as ACK, NAK and DTX, where $p_{A \rightarrow B}$ stands for the probability that A is detected as B. Finally, no retransmission is caused for the detected ACK, but retransmission is triggered by detecting either NAK or DTX.

When the resource is successfully assigned and the transport block is correctly transmitted on PHY layer, the transmission of MAC SDU succeeds regardless of retransmission. In case of successful resource assignment and erroneous transmission of PHY transport block, as well as in case of resource assignment failure, the transmission on MAC layer is correct, if retransmission on PHY layer succeeds, but the transmission is erroneous, if retransmission fails or no retransmission is carried out. All the green paths representing a correct transmission of MAC SDU are collected into one group and all the red paths for an erroneous transmission are categorized into the other group.

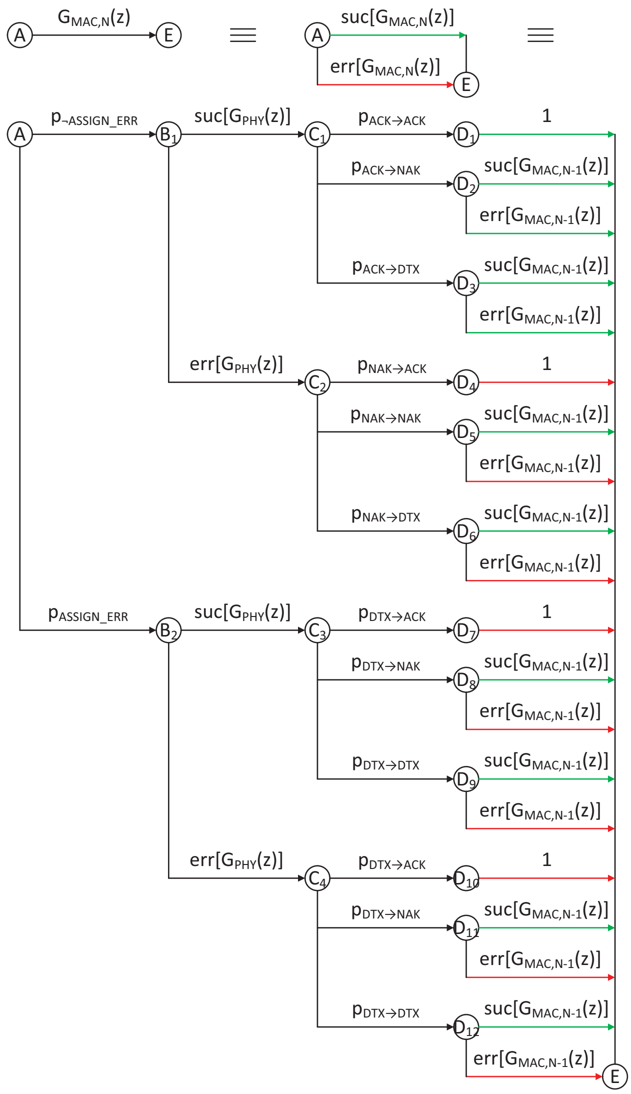


Fig. 3. SFG for MAC layer: recursion

As exit condition for the recursion, the SFG for the transmission of MAC SDU within only 1 transmission on PHY layer is shown in Fig. 4, being the simplest case of Fig. 3.

B. *Moment Generating Function of MAC Layer*

The MGF for the exit condition is derived from the SFG. The part corresponding to a correct transmission of MAC SDU and the erroneous part are given as follows, respectively.

$$G_{MAC,1}(z) = suc[G_{MAC,1}(z)] + err[G_{MAC,1}(z)] \qquad (1)$$

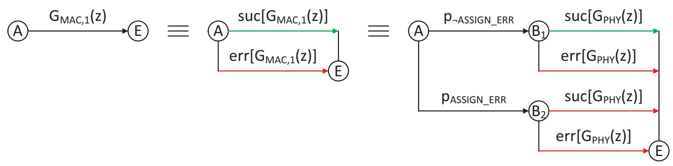


Fig. 4. SFG for MAC layer: exit condition

$$suc[G_{MAC,1}(z)] = p_{\neg ASSIGN\_ERR} \cdot suc[G_{PHY}(z)] \quad (2)$$

$$\begin{aligned} err[G_{MAC,1}(z)] &= p_{\neg ASSIGN\_ERR} \cdot err[G_{PHY}(z)] \\ &+ p_{ASSIGN\_ERR} \cdot (suc[G_{PHY}(z)] \\ &+ err[G_{PHY}(z)]) \end{aligned} \quad (3)$$

The MGF for the recursion can also be obtained from the SFG straightforwardly but is omitted here.

C. Resource Consumption, Error Ratio and Spectral Efficiency

Without loss of generality, assume that $G_{Layer}(z)$ is the MGF of a certain layer and consists of a success part $suc[G_{Layer}(z)]$ and a failure part $err[G_{Layer}(z)]$. p_i in Eq. 4 is the probability that an SDU with a given size is transmitted on this layer in the i^{th} radio channel state. $\#PRBP_i$ represents the number of physical resource block pairs (PRBPs) consumed for such a transmission. e_i in Eq. 5 is the conditional probability that the transmission is performed but fails and $(p_i - e_i)$ in Eq. 6 is the conditional probability that the transmission is carried out and succeeds. $\frac{e_i}{p_i}$ is the error rate of such a transmission, while $\frac{p_i - e_i}{p_i}$ is the success rate.

$$\begin{aligned} G_{Layer}(z) &= suc[G_{Layer}(z)] + err[G_{Layer}(z)] \\ &= \sum_{i=1}^{N} (p_i \cdot z^{\#PRBP_i}) \end{aligned} \quad (4)$$

$$err[G_{Layer}(z)] = \sum_{i=1}^{N} (e_i \cdot z^{\#PRBP_i}) \quad (5)$$

$$suc[G_{Layer}(z)] = \sum_{i=1}^{N} ((p_i - e_i) \cdot z^{\#PRBP_i}) \quad (6)$$

Consequently, p_i is the probability for consumption of resource $\#PRBP_i$, Eq. 7. p_i is the probability for error ratio equal to $\frac{e_i}{p_i}$, Eq. 8.

$$\begin{aligned} Prob[ResourceConsumption = \#PRBP_i] = p_i, \\ i = 1, 2, ..., N \end{aligned} \quad (7)$$

$$Prob[ErrorRatio = \frac{e_i}{p_i}] = p_i, i = 1, 2, ..., N \quad (8)$$

Recall that spectral efficiency is the number of successfully transmitted bits normalized by the consumed resource in time and in bandwidth [3]. Payload represents the number of transmitted bits and, therefore, $Payload \cdot (\frac{p_i - e_i}{p_i})$ is the number of successfully transmitted bits. $\#PRBP_i$ represents the number of consumed PRBPs, each of which occupies 1 ms in time domain and 180 KHz in frequency domain. Hence, $\#PRBP_i \cdot (10^{-3} \cdot 180 \cdot 10^3)$ is the consumed resource in $s \cdot Hz$. Consequently, p_i is the probability for spectral efficiency equal to $\frac{payload \cdot \frac{(p_i - e_i)}{p_i}}{\#PRBP_i \cdot (10^{-3} \cdot 180 \cdot 10^3)}$, Eq. 9.

$$\begin{aligned} &Prob[SpectralEfficiency[\frac{bit}{s \cdot Hz}] \\ &= \frac{payload \cdot \frac{p_i - e_i}{p_i}}{\#PRBP_i \cdot (10^{-3} \cdot 180 \cdot 10^3)}] = p_i, \\ &i = 1, 2, ..., N \end{aligned} \quad (9)$$

D. Probabilistic Radio Channel States in Relay Enhanced Scenario

Referring to the relay enhanced urban macro-cell (UMa) scenario [9] shown in Fig. 5, one base station (BS), which performance parameters are evaluated, is placed in the center of the scenario and three antennas are employed by the BS at 30°, 150° and 270°, respectively. Single relay node (RN) per cell is placed 0.81 cell radius away from the center BS in antenna boresight direction. According to [5] this appears to be a close to optimum position. One tier of six BSs each with three RNs is taken into account for interference consideration.

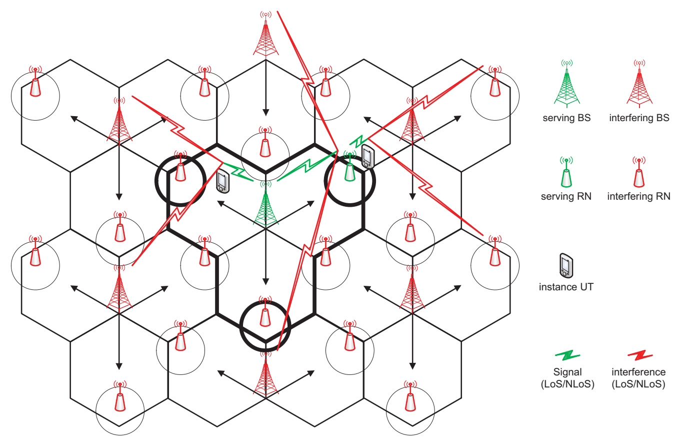


Fig. 5. Relay enhanced UMa scenario

According to the UMa channel model [9], path loss is specified for both line of sight (LoS) and non LoS (NLoS) radio propagation conditions, where the probability of LoS is defined as a function of distance. As illustrated in Fig. 5, a RN on downlink besides the signal from its donor BS also simultaneously receives cochannel interferences from up to six neighbor BSs. A user terminal (UT) on downlink is served either by a BS or by a RN and is interfered either by up to six neighbor BSs or by up to twenty neighbor RNs. Radio propagation conditions, both between serving transmitter and receiver and between interfering transmitters and receiver, may be either LoS or NLoS.

In one hop model there are 2^7 radio channel states for access link between BS and UT. In two hop model there are 2^{21} radio channel states for access link between RN and UT, whilst the backhaul link between BS and RN has the same number of states as the access link in one hop cell. The analysis method presented here is also applicable to scenarios with multiple RNs per cell, but requires excessive computation power owing to the much increased number of possible interferers. It is still a challenge to calculate the performance of system with more than one RN per cell.

For backhaul link radio propagation condition between serving BS and RN is assumed to be LoS, while, as a result of careful RN placement, conditions between interfering BSs and RN are NLoS. For access link NLoS radio propagation conditions between interfering RNs and UT are assumed for the interfering RNs, which are quite far away from the UT, so that LoS probability is below 5%.

E. LTE Radio Frame for Relay Enhanced System

The LTE radio frame for downlink transmission in frequency division duplex (FDD) mode and various types of physical resource block pairs (PRBPs) are shown in Fig. 6. Following [10], the physical broadcast channel (PBCH) is located in the first subframe and within the 6 resource blocks (RBs) in the middle of frequency band. The synchronization sequences are located in the first and the sixth subframes and within the minimal bandwidth. The first 3 symbols of PRBP are occupied by downlink control channels, which include the physical control format indicator channel (PCFICH), the physical hybrid-ARQ indicator channel (PHICH) and the physical downlink control channel (PDCCH). In case of 1 antenna port, 8 resource elements (REs) per PRBP are reserved by cell specific reference symbols.

Accounting for relay enhanced LTE system [11], only multicast broadcast single frequency network (MBSFN) subframes configured by relay subcells, e.g. the 2^{nd}, 4^{th}, 7^{th} and 9^{th} subframes in Fig. 6, can be utilized for backhaul links to transmit data from base station (BS) to relay nodes (RNs). Non MBSFN subframes are available for physical downlink shared channels (PDSCHs), which are shared by BS and RNs to serve user terminals (UTs) over access links. Some RBs in MBSFN subframes, e.g. the 2^{nd} of the 6 RBs in the middle frequency band in Fig. 6, are occupied for the relay physical downlink control channels (R-PDCCHs) and some others, e.g. the 4^{th} and 5^{th} RBs in the middle frequency band, are available for the relay physical downlink shared channels (R-PDSCHs), which are utilized for data transmission over backhaul link from BS to RNs. The other RBs in MBSFN subframes cannot be used for access links between RNs and their own UTs, but merely for access links from BS to its own UTs.

In this work, all the colored REs for signaling are regarded as overhead and only the remaining uncolored REs are

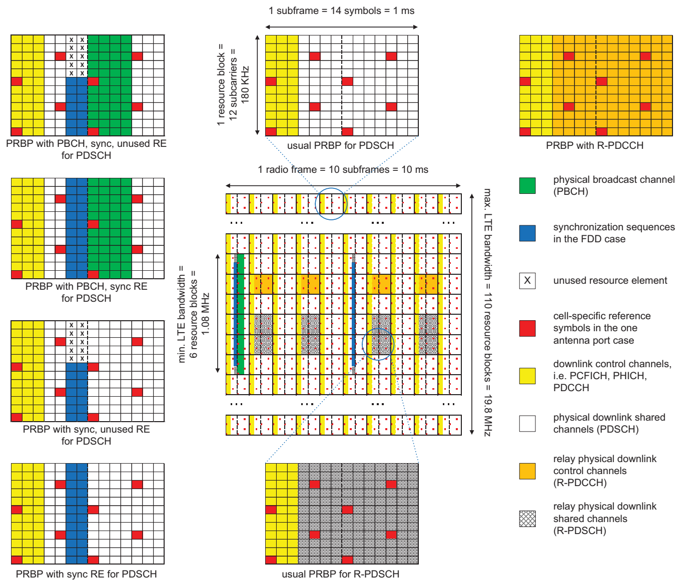


Fig. 6. LTE radio frame for relay enhanced system

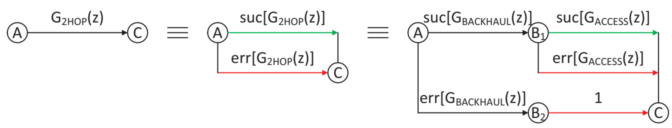


Fig. 7. SFG for two hops

assumed available for downlink data transmission on either backhaul links or access links.

F. Signal Flow Graph Model for Two Hop Transmission

The SFG specifying two hop transmission based on decode and forward relaying is shown in Fig. 7. As a first step, data is transmitted either correctly or erroneously over backhaul link. If data is successfully decoded from the first hop, then it is forwarded to the second hop. In a final step, data transmission over access link is either correct or erroneous. As a result, the whole two hop transmission is correct, if and only if the transmission succeeds both over backhaul link and over access link, while erroneous, so long as the transmission fails either over backhaul link or over access link. The only green path represents a correct two hop transmission and therefore belongs to one group. The other two red paths represent a two hop transmission in error and thus belong the other group.

G. Moment Generating Function for Two Hop Link

The MGF for two hop transmission is derived from the SFG. The part corresponding to a correct transmission of SDU on a certain layer and the erroneous part are given as follows, respectively.

$$G_{2HOP}(z) = suc[G_{2HOP}(z)] + err[G_{2HOP}(z)] \qquad (10)$$

$$suc[G_{2HOP}(z)] = suc[G_{BACKHAUL}(z)] \cdot suc[G_{ACCESS}(z)] \quad (11)$$

$$err[G_{2HOP}(z)] = suc[G_{BACKHAUL}(z)] \cdot err[G_{ACCESS}(z)] + err[G_{BACKHAUL}(z)] \quad (12)$$

IV. Performance Calculation

The urban macro-cell (UMa) scenario [9] is considered here for example calculation. Parameters for the relay enhanced UMa scenario are given in Table I.

TABLE I
PARAMETERS OF RELAY ENHANCED UMA SCENARIO [9]

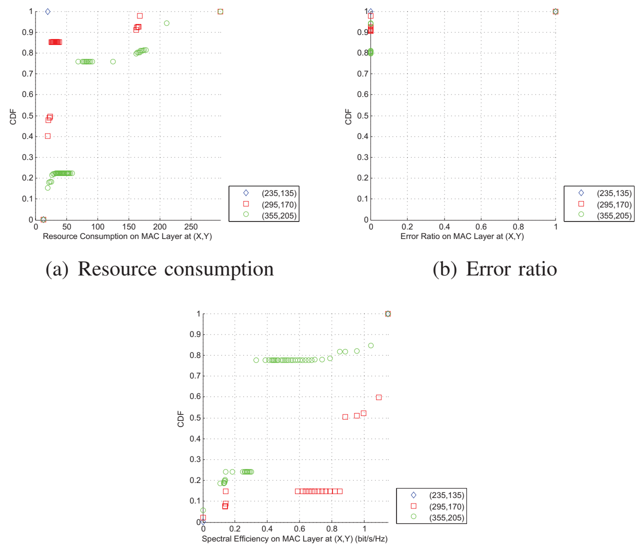
scenario	**urban macro-cell (UMa)**
layout	hexagonal grid
inter-site distance	500 m
BS antenna height	25 m, above rooftop
# BS antennas	1 tx
total BS transmit power	46 dBm for 10 MHz
BS antenna down tilt angle	12°
RN antenna height	10 m, below rooftop
# RN antennas	1 rx, 1 tx
total RN transmit power	27 dBm for 10 MHz
RN antenna down tilt angle	omni-directional
UT antenna height	1.5 m
# UT antennas	1 rx
minimum distance between UT and BS	$\geq$ 25 m
carrier frequency	2 GHz
channel model	UMa (LoS, NLoS)
BS noise figure	5 dB
RN noise figure	5 dB
UT noise figure	7 dB
BS antenna gain (boresight)	17 dBi
RN antenna gain	0 dBi
UT antenna gain	0 dBi
thermal noise level	-174 dBm/Hz

It is assumed for example calculation, that a radio link control (RLC) service data unit (SDU) comprises 1024 bytes and is segmented into 2 RLC protocol data units (PDUs). RLC header, medium access control (MAC) header and physical (PHY) cyclic redundancy check (CRC) are 4 bytes, 3 bytes and 3 bytes, respectively. A user terminal (UT) is associated to either the base station (BS) or the relay node (RN), dependent on the best received mean signal to interference plus noise ratio (SINR). A maximum of one retransmission is assumed permitted for the hybrid automatic repeat request (H-ARQ) protocol. Neither resource assignment failure nor feedback misdetection is taken into account in the example considered here. Full buffer traffic on downlink and resource fair scheduling by BS are assumed for evaluation of cell spectral efficiency.

In the following, results are presented for three system types, namely system with BS only, relay enhanced system with wired backhaul link, and relay enhanced system with wireless backhaul link. As an example, performance results of Matlab calculation for the MAC layer are given here. In the multi-cell scenario a 5 m $\times$ 5 m grid is introduced over an 800 m $\times$ 800 m center area of the scenario. Resource consumption, error ratio and spectral efficiency are assessed from cumulative distribution function (CDF) for any grid point, expected value for each grid point, CDF over all grid points in a center cell, as well as expected value and percentile for a center cell. The respective results for the other layers are available but omitted here.

A. CDF for any Location

CDFs of resource consumption, error ratio and spectral efficiency at the example locations for a relay enhanced system with wireless backhaul are shown in Fig. 8(a) - 8(c). Numerical results on MAC layer are calculated for the example locations in the northeast relay cell of the central site, namely blue, green and red locations located in the center of the relay cell, at the edge of the relay cell and in the middle between the center and the edge, respectively.



(a) Resource consumption

(b) Error ratio

(c) Spectral efficiency

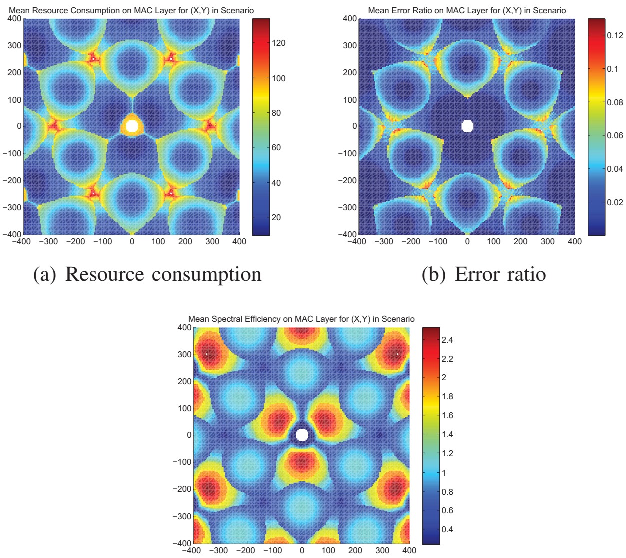
Fig. 8. CDFs of performance criteria on MAC layer at (X, Y) coordinates

Due to best radio channel conditions, the UT at blue location consumes fewest resources, experiences lowest error ratio and achieves highest spectral efficiency. Due to worst channel conditions, the UT at green location consumes most resources, experiences highest error ratio and achieves poorest spectral efficiency.

B. Scenario Map

Multi-cell scenario maps of resource consumption, error ratio and spectral efficiency on MAC layer for a relay enhanced system with wireless backhaul are shown in Fig. 9(a) - 9(c).

Edges between donor cell and relay cells are clearly visible. Relay cell edge users consume more resources, experience higher error ratio and suffer from lower spectral efficiency than donor cell edge users. The reason is that a UT is associated to the radio access point (RAP), i.e. the BS or the RN, from



(a) Resource consumption

(b) Error ratio

(c) Spectral efficiency

Fig. 9. Scenario maps of performance criteria on MAC layer

which it receives the best mean SINR, and transmission on backhaul link consumes extra resources and causes more errors for two hop UTs.

A relay subcell and a donor subcell cover 51.34% and 48.66% of a cell, respectively. A RN serves a larger percentage of the cell than a BS, since cochannel interference between RNs is lower than that between BSs.

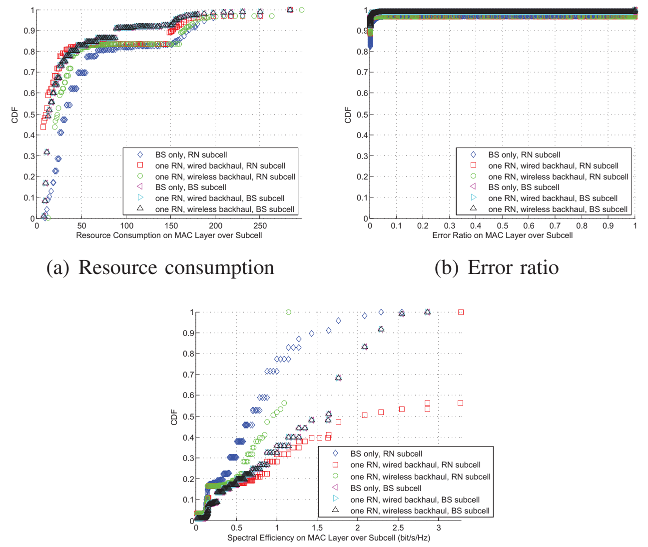
Areas in antenna boresight and nearby BS in donor cells, and center areas in relay cells consume least resources, experience lowest error ratio and benefit from highest spectral efficiency ≥ 2 bit/s/Hz and ≥ 1.2 bit/s/Hz, respectively, thanks to good radio channel conditions.

Bordering areas of three adjacent relay cells belonging to three neighbor sites, e.g. areas close to central points located one cell radius, i.e. 333 m, apart the center BS at 0°, 60°, 120°, 180°, 240° and 300°, respectively, consume substantial radio resources in the cell, experience highest error ratio $\geq 5\%$ and suffer from worst spectral efficiency ≤ 0.5 bit/s/Hz owing to strong cochannel interference between adjacent relay cells and between neighbor sites.

Circular areas close by and around BSs suffer from worst spectral efficiency ≤ 0.5 bit/s/Hz and consume much resources due to strong cochannel interference between adjacent cells of a site. However, they experience an acceptable error rate.

C. CDF for a Cell

Fig. 10(a) - 10(c) show CDFs of resource consumption, error ratio and spectral efficiency for various parts of a cell. Probability distributions of performance parameters are aggregated over all grid points in relay cell for blue, red and green cases, and over donor cell for pink, turquois and black cases. A system without relaying is evaluated for blue and pink cases. A relay enhanced system with quasi wired backhaul is investigated for red and turquois cases, where backhaul links are not accounted for but only access links are taken into account. A relay enhanced system with wireless backhaul is assessed for green and black cases, where two hop transmission including backhaul and access links is taken into consideration.



(a) Resource consumption

(b) Error ratio

(c) Spectral efficiency

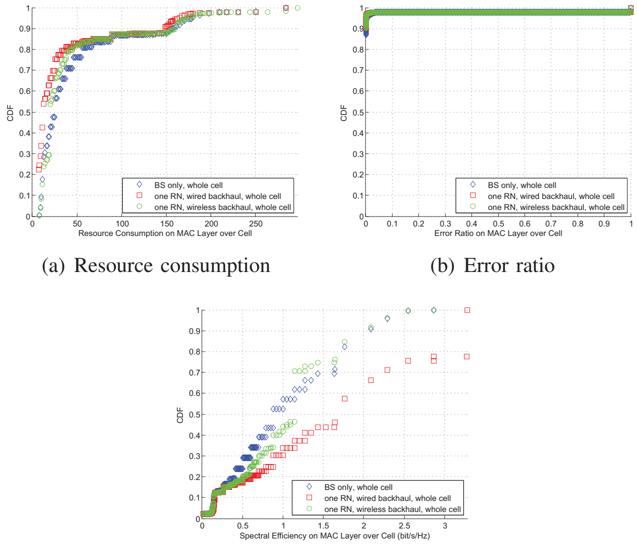
Fig. 10. CDFs of performance criteria on MAC layer in subcells

Blue case consumes much radio resource, experiences the highest error ratio and suffers from the lowest spectral efficiency due to bad radio channel conditions. Compared to blue case, red case significantly decreases resource consumption, has similar error ratio and substantially increases spectral efficiency, because radio channel conditions over access links are substantially improved and some gain is obtained. In comparison to red case, resource consumption of green case is higher, error ratio is comparable and spectral efficiency is lower, since data transmission over wireless backhaul link costs radio resource and some gain obtained over access links is lost. Compared with blue case, green case reduces resource consumption, has similar error ratio and raises spectral efficiency, because the gain obtained over access links is more than the gain lost over backhaul link and some considerable gain is still retained.

Pink, turquois and black cases overlap each other as expected. Among blue, red and green cases, red case is the closest one to turquois case, because the association strategy of the best mean SINR results in similar radio channel conditions between relay cell and donor cell.

Fig. 11(a) - 11(c) show CDFs of resource consumption, error ratio and spectral efficiency over cell for different systems. Blue represents the system without relaying, red represents the relay enhanced system with wired backhaul, and green represents the relay enhanced system with wireless backhaul.

As CDFs of resource consumption, error ratio and spectral efficiency over cell are aggregated from CDFs over relay cell and CDFs over donor cell, green has less resource consump-



(a) Resource consumption

(b) Error ratio

(c) Spectral efficiency

Fig. 11. CDFs of performance criteria on MAC layer in cell

tion, similar error ratio and higher spectral efficiency compared to blue, but more resource consumption, comparable error ratio and lower spectral efficiency compared to red.

D. Cell (Edge User) Spectral Efficiency

Expected values of resource consumption, error ratio and spectral efficiency are regarded as performance measures for cell, while the 95^{th} percentiles of both resource consumption and error ratio, as well as the 5^{th} percentile of spectral efficiency are performance measures for cell edge users, only. Expected value and the 5^{th} percentile of spectral efficiency are addressed in [3] as cell spectral efficiency (CSE) [bit/s/Hz/cell] and cell edge user spectral efficiency (CE-USE) [bit/s/Hz], respectively.

Results are given in Table II showing resource consumption rounded to two decimals, and error ratio and spectral efficiency rounded to four decimals. By employing one RN per cell and wireless backhaul in LTE system, thereby modeling relay enhanced cells, both CSE and CE-USE are raised from 1.0834 and 0.1268 by 2.32% and 3.31% to 1.1085 and 0.1310, respectively, compared to a system without relays. By employing wired backhaul, therby modeling picocell enhanced donor cells, both CSE and CE-USE are significantly raised from 1.0834 and 0.1268 by 59.63% and 9.62% to 1.7294 and 0.1390, respectively, compared to a system without relays.

V. Conclusion

An analytical framework is presented to evaluate cumulative distribution functions (CDFs) of resource consumption, error ratio and spectral efficiency specific to relay cell, donor cell and whole cell, respectively, in relay enhanced and picocell enhanced 3GPP LTE system. Decode and forward relaying is modeled and evaluated with respect to its contribution to interesting performance measures for a cell in a cellular scenario. Results are presented for one relay per cell.

TABLE II
Expected value and percentile of performance parameter

system	area	resource		error		efficiency	
		mean	95%	mean	95%	mean	5%
BS only	RN subcell	60.66	198.00	0.0324	0.0164	0.7716	0.1156
	BS subcell	35.84	156.00	0.0109	0.0000	1.4124	0.1470
	whole cell	48.59	180.00	0.0219	0.0045	1.0834	0.1268
one RN wired backhaul	RN subcell	40.94	168.00	0.0326	0.0006	2.0298	0.1361
	BS subcell	35.84	156.00	0.0109	0.0000	1.4124	0.1470
	whole cell	38.46	165.00	0.0221	0.0000	1.7294	0.1390
one RN wireless backhaul	RN subcell	53.94	181.00	0.0326	0.0006	0.8203	0.1263
	BS subcell	35.84	156.00	0.0109	0.0000	1.4124	0.1470
	whole cell	45.13	175.00	0.0221	0.0000	1.1085	0.1310

It is found that the relay enhanced LTE system, although consuming radio resources for data transmission over wireless backhaul link, contributes to improve both cell spectral efficiency (CSE) and cell edge user spectral efficiency (CE-USE). It is also shown that the picocell enhanced system is useful to further increase system performance. It remains to explore under the basic conditions of the IMT-Advanced evaluation process [9], to what degree relay enhanced and picocell enhanced LTE system can meet or even exceed the IMT-Advanced requirements [3] under single input single output (SISO) transmission and reception.

The analytical framework introduced can be extended to evaluate CSE of LTE system enhanced with more than one relay per cell and this is the next step of our intended work.

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