The Influence of Subchannel Diversity on the Performance of OFDMA Systems based on IEEE 802.16

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Abstract—**The performance of a mobile radio system based on OFDMA is determined by the type of mapping of logical subchannels, which are allocated for data transmissions, onto the frequency channel. The mapping scheme affects the diversity both within and between the OFDMA subchannels. This paper provides a performance comparison of different subchannel mappings from MAC point of view by means of stochastic event-driven simulations. The tradeoff between the effects of mapping subchannels onto distributed and adjacent subcarrier sets are investigated in detail. The effects are robust SNR estimation and possible throughput gains due to frequency adaptive scheduling, respectively.**

I. Introduction

Orthogonal Frequency Division Multiple Access (OFDMA) is considered to be one of the key technologies for future broadband radio systems such as WiMAX [1] which recently became a member of the International Mobile Telecommunications-2000 (IMT-2000) family of systems, or the 3GPP Long Term Evolution (LTE) [2]. IMT-Advanced next generation mobile networks will probably also employ OFDMA technology. It has been shown that exploiting multiuser diversity can significantly increase the system capacity [3]. Nevertheless, these expected capacity gains can be compensated by transmission error due to uncertainty in the channel state estimation.

The performance evaluation presented in this paper addresses the tradeoff between diversity exploitation and accuracy in the SNR estimation for different types of mapping of logical OFDMA subchannels onto subcarriers in the frequency channel resulting in different degrees of subchannel diversity.

This paper extends the work presented in [4], in which only the two extreme cases of maximum diversity within and between OFDMA subchannels have been compared for IEEE 802.16 [5]. The focus is on the interaction between physical layer (PHY) and medium access control (MAC), and the performance evaluation is conducted by means of stochastic event-driven simulations.

The rest of the paper is organized as follows. First, a short description of the channelization in OFDMA systems is given in Section II. In Section III, the diversity within and between OFDMA subchannels is determined for different mapping schemes. The performance evaluation is presented in Section IV. Finally, the paper ends with conclusion.

II. OFDMA Subchannels

In an OFDMA system, the subcarriers are normally grouped into logical subchannels. A subchannel defines the granularity of the resource allocation in the frequency domain. In the time domain, the granularity is determined by the slot length. Within an OFDMA subchannel, the same modulation and coding scheme (PHY mode) is used for each subcarrier.

The design of these subchannels has a crucial impact on the performance of an OFDMA system. The performance is determined by the diversity of the subcarriers within a subchannel and the diversity between the subchannels. Furthermore, the accuracy of the SNR estimation at the transmitter, which depends on the Doppler shift, is of significant relevance.

The quality of an OFDMA subchannel from MAC point of view is described by an effective SNR level. This SNR level can be determined in different manners [6]. In the following the effective SNR is the mean SNR of all subcarriers of a subchannel which is a common assumption, e.g. in [7]. Subcarriers with bad channel conditions can be compensated by subcarriers with good conditions within an OFDMA subchannel due to channel coding [8].

III. Determination of Subchannel Diversity

The power attenuation in a radio channel can be described as a superposition of mean pathloss, shadowing, and multipath fading [9]. Since the impact of pathloss and shadowing coincide for all OFDMA subchannels independent of the mapping onto the frequency channel, the subchannel diversity can be completely determined by the investigation of multipath fading (i.e., fast fading).

To quantify the diversity of an OFDMA subchannel, a metric has been proposed in [10]. It is defined as the variance of the mean fading level of the subcarriers in an OFDMA subchannels normalized by the variance of the subcarrier fading. It is given in (1), where γ_i denotes the fading level on subcarrier i, M the number of subcarriers per subchannel, and σ^2 the variance of the subcarrier fading.

$$D(X) = \frac{1}{\sigma^2}\mathrm{Var}\left(\frac{1}{M}\sum_{i=1}^{M}\gamma_i\right) \tag{1}$$

In the extreme case when all subcarriers of a subchannel have maximum correlation, i.e., all have always the same fading level, the expression reduces to $D = 1$. If all subcarriers are independent of each other then $D = 1/M$. In general, the diversity metric is between these two extremes. Please note, a value of this diversity metric decreases if the diversity increases.

In this work, this diversity metric is used to quantify both the diversity within an OFDMA subchannel (D_{intra}) and the diversity between the usable OFDMA subchannels (D_{inter}).

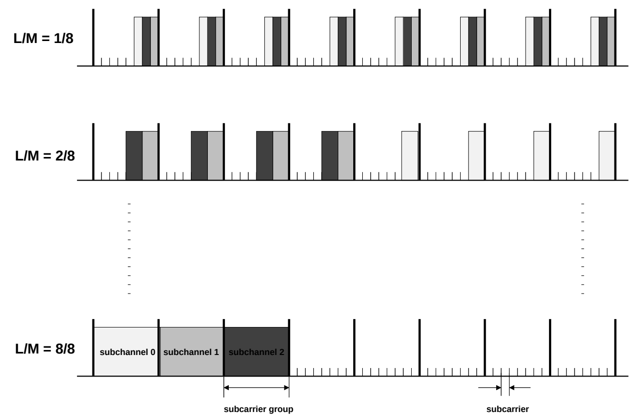

Fig. 1. Mapping of logical subchannels onto physical resources in the frequency domain (M: subcarriers per subchannel, L: subcarriers from the same subband in a sunchannel)

Fig. 1 shows the mapping of logical OFDMA subchannels onto subcarriers in the frequency domain as it is used in this work. The frequency channel is assumed to be subdivided into subbands with flat fading, i.e., one subband does not exceed the coherence bandwidth of the frequency channel. In this figure, the channel is exemplary divided into eight subbands, and eight OFDMA subchannels are mapped onto these sub-channels. A subchannel comprises up to eight subcarriers of one subband.

The calculation of the metric for the diversity within a single OFDMA subchannel is given in (2). The number of subcarriers within a subband which contribute to the subchannel is L, and these subcarriers have always the same fading level. Hence, M/L different uncorrelated subbands are used within a sub-channel (M is the number of subcarriers within a subchannel).

$$D_{intra}(X) = \frac{1}{\sigma^2}\mathrm{Var}\left(\frac{1}{M}\sum_{i=1}^{M/L} L\cdot\gamma_i\right) \tag{2}$$

The following equations show how (2) can be reduced when the subband fading processes are independent, which means that the covariance is zero.

$$\begin{aligned} D_{intra}(X) &= \frac{1}{\sigma^2}\frac{L^2}{M^2}\mathrm{Var}\left(\sum_{i=1}^{M/L}\gamma_i\right) && (3)\\ &= \frac{1}{\sigma^2}\frac{L^2}{M^2}\left(\frac{M}{L}\sigma^2\right) = \frac{L}{M} && (4)\end{aligned}$$

Let γ_{s_i} be the effective fading level of OFDMA subchannel i and σ_s^2 the variance of the effective fading. Hence, the metric for the diversity between the different OFDMA subchannel is given by (5). The system consists of N subchannels.

$$D_{inter}(X) = \frac{1}{\sigma_s^2}\mathrm{Var}\left(\frac{1}{N}\sum_{i=1}^{N}\gamma_{s_i}\right) \tag{5}$$

The reduction of this equation is given in the following.

$$\begin{aligned} D_{inter}(X) &= \frac{1}{\sigma_s^2}\mathrm{Var}\left(\frac{1}{N}\sum_{i=1}^{\frac{N}{M/L}}\frac{M}{L}\cdot\gamma_{s_i}\right) && (6)\\ &= \frac{1}{\sigma_s^2}\frac{1}{N^2}\frac{M^2}{L^2}\left(\frac{NL}{M}\sigma_s^2\right) = \frac{N}{LM} && (7)\end{aligned}$$

Corresponding to Fig. 1, M/L OFDMA subchannels have the same subband pattern. Therefore, these subchannels have also always the identical effective fading level. Subchannels with different subband patterns are independent of each other because the patterns are orthogonal.

The resulting ratio of diversity within a single OFDMA subchannel and the diversity between all usable subchannels is given in (8). If a subchannel consists only of subcarriers from a single subband, the inter subchannel diversity is maximal and the intra subchannel diversity is minimal. Corresponding to the introduced metric, D_{inter} is minimal and D_{intra} is maximal.

$$\frac{D_{inter}(X)}{D_{intra}(X)} = \frac{N}{L^2} \tag{8}$$

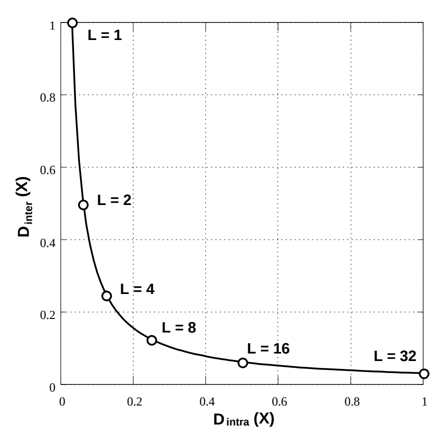

Fig. 2. Diversity metric evaluation

The relation between $D_{intra}(X)$ and $D_{inter}(X)$ is depicted in Fig. 2. The quantity of both OFDMA subchannels (N)

and uncorrelated subbands is 32. Furthermore, each subband comprises 32 subcarriers (M). The calculation shows the tradeoff between the possibility to efficiently exploit multiuser diversity due to small D_{inter} and accurate SNR estimation due large magnitudes of D_{intra}.

IV. Performance Evaluation

A. System Description

The parameter settings of the simulated OFDMA system are given in Table I. The MAC frame of the centrally controlled system has a simplified structure based on IEEE 802.16 with a length of 1 ms. A single time slot with a length of two symbols (201.6 μs) is reserved for downlink transmissions within each MAC frame. This means that 32 resource elements, 32 OFDMA subchannels within the downlink time slot, can be assigned by the scheduler. A resource element has a capacity of one, two or three MAC protocol data units (MPDUs) depending on the used PHY mode.

TABLE I
OFDMA system parameter (based on IEEE 802.16)

MAC frame length	1.0 ms
OFDMA subchannels	32
MPDU size	18 byte
Bandwidth	20 MHz
Subcarriers	2048 = $32 \times (48 + 16)$
Data subcarriers per subchannel	48
Symbol length	100.8 μs
Slot length	2×100.8 μs
DL slots	1
PHY modes	QPSK3/4, 16QAM3/4, 64QAM3/4

The packet error rate (PER) depending on SNR has been determined under the assumption of an AWGN channel based on the equations given in [11]. A constant gain of 2 dB is assumed for the channel coding. The resulting PER curves are shown in Fig. 3. The figure also shows the SNR ranges for the PHY modes with maximum PER of 10 %.

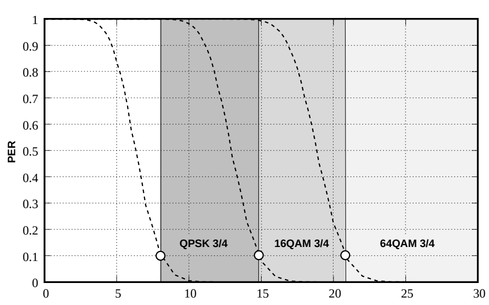

Fig. 3. PER curves and PHY mode ranges

The time correlated fading levels for the subbands, which are mutually independent, are generated with Jakes' model [12]. It is assumed that the duration between SNR measurement and resource scheduling is 1 ms, corresponding to the length of a MAC frame.

Resource elements are allocated to data connections at the beginning of each MAC frame by the following strategy. From the set of all connection with at least one MPDU in the transmission queue and the set of all available resources, the pair with the maximum SNR is selected. The selected resource is assigned to the according connection and removed from the set of available resources. Depending on the estimated SNR level for fixed transmission power per allocation, the PHY mode is selected and the according number of MPDUs is removed from the transmission queue. This procedure is repeated until either all MPDUs are transmitted or all resource elements are allocated.

Retransmission due to MPDU errors are handled by an SR-ARQ scheme. Corrupted MPDUs are assumed to be retransmitted in the next MAC frame.

B. Simulation Scenario

The scenario consists of one base station (BS) and 32 mobile terminals (MTs), each with a downlink data connection. The mean SNR level without fading is 15 dB for each MT. The traffic load is modeled by Poisson streams. Simulations have been conducted for Doppler shifts f_m of 100 and 200 Hz, which correspond to terminal velocities of 21.6 and 43.2 km/h at 5 GHz. The coherence time T_C for a flat fading channel (corresponding to the usage of a single subband per OFDMA channel) is approximated by [13]:

$$T_C \approx \frac{9}{16\pi f_m} \tag{9}$$

With this definition, the correlation times in the simulations amount to 1.8 ms and 0.9 ms, respectively. Hence, the time between SNR measurement and MPDU transmission is larger than the the coherence time with f_m = 200 Hz. This means that the uncertainty in the SNR estimation is imprecise with the application of the subchannel mapping with maximum inter subchannel diversity. That can cause a significant performance degradation in terms of delay and throughput.

C. Simulation Results

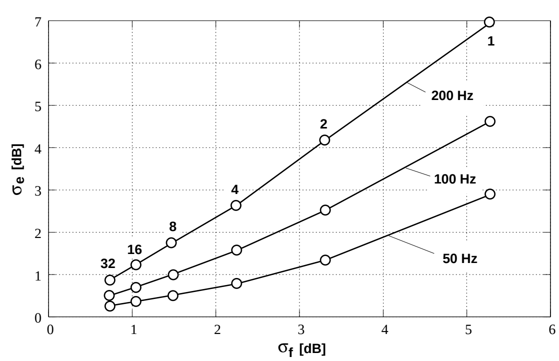

Fig. 4. Dependency between standard deviation of fading level (σ_f) and estimation error (σ_e) for different OFDMA subchannel mapping schemes (the conditional estimation accuracy depending on the current fading level is not investigated in detail)

Fig. 4 shows the standard deviation of the SNR estimation error depending on the standard deviation of the effective subchannel fading level. The latter is, as explained in Section III, determined by the OFDMA subchannel mapping onto subbands. The SNR estimation error reduces when the subcarriers of a subchannel are distributed in the frequency channel which results in an increased intra subchannel diversity.

Fig. 5 shows the mean estimated SNR depending on traffic load and OFDMA subchannel mapping for 100 Hz Doppler shift. This figures reveals the possibilities to increase the SNR by the use of subchannel mappings with large degrees of inter subchannel diversity (small number of used subbands per subchannel). The SNR difference between the subchannel mapping with minimum and maximum inter subchannel diversity amounts to 5 dB at low traffic loads. When the traffic load is increased, the SNR gain due to the exploitation of subchannel diversity is reduced because also resource with low quality are used by a data connection for MPDU transmissions. The larger the inter subchannel diversity is, the larger is also the extent of this SNR reduction.

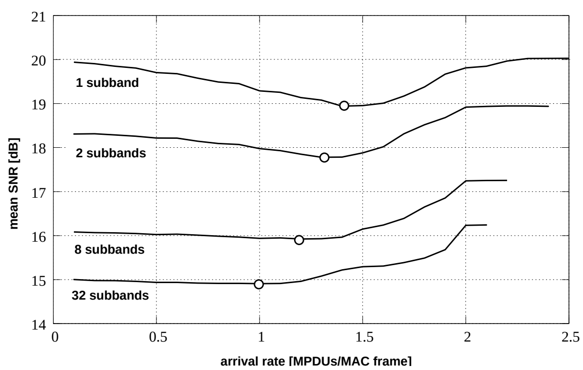

Fig. 5. Mean SNR with 100 Hz Doppler shift

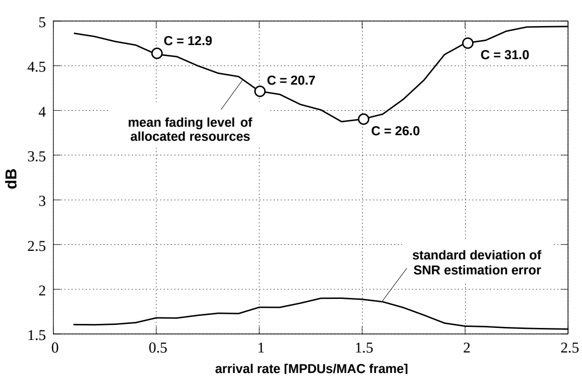

Fig. 6. Standard deviation of SNR estimation error and mean fading level of scheduled resources with 100 Hz Doppler shift for mapping of a subchannel onto a single subband; C is the mean number of data connections with MPDUs in the transmission queue

After reaching a minimum, which is marked in the figure, the SNR level again grows when the traffic load is further increased. This is based on the fact that with high traffic loads also the probability that more data connections have MPDUs in the transmission queue at the same time increases. This means that a more efficient resource utilization can be achieved due to the exploitation of multiuser diversity. When the number of data connections with MPDUs in the transmission queues is small at low traffic loads, these connections are scheduled more or less independently of the channel quality.

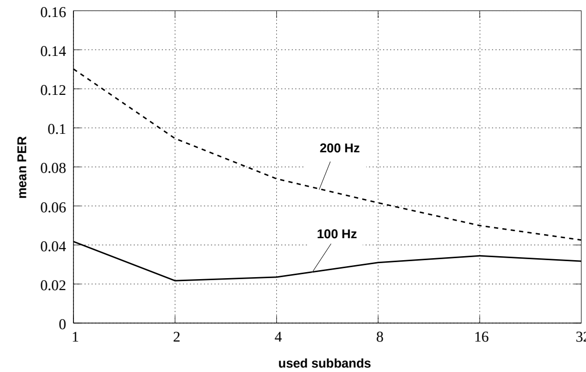

Fig. 7. Mean PER with arrival rate of one MPDU per MAC frame

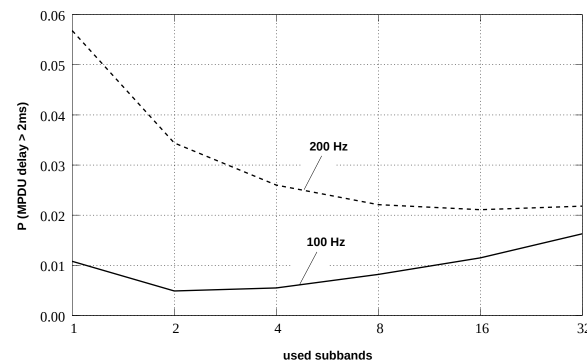

Fig. 8. MPDU delay with arrival rate of one MPDU per MAC frame

Fig. 6 illustrates the SNR estimation accuracy (lower graph). The curve shows the standard deviation of the estimation error for the mapping of a subchannel onto a single subband with 100 Hz Doppler shift. The accuracy reduces with increasing traffic load because more and more subchannels with lower fading levels (upper graph) have to be used to transmit all MDUs. After reaching a point of maximum uncertainty, the variance of the SNR estimator decreases again when the traffic load is further increased because the fading level increase again due to the effect explained above.

The relation of both curves is explained by the fact that the duration of fades increases with the level of fading [12]. In other words, the higher the fading level of the allocated resources (upper graph), the more precise the SNR estimator performs (lower graph). Apparently, the operation point of maximum variance of the SNR estimator matches the point of the minimum mean fading level.

Due to uncertainty in the SNR estimation, the PER grows when the Doppler shift is increased. This can be seen in Fig. 7 for a traffic load of one MPDU per MAC frame (144 kbit/s) for each connection. The results reveal that an optimum mapping exists for OFDMA subchannels which minimizes the mean PER. The interesting fact is here, the optimum mapping does not correspond to the mapping with the smallest variance in the

SNR estimation. This is based on the fact that with the use of subchannel mappings with large degrees of inter subchannel diversity the mean SNR can be significantly increased due to the exploitation of multiuser diversity. But as shown in Fig.1, large inter subchannel diversity also implies low intra subchannel diversity which increases the uncertainty on the SNR estimation.

The impact on the MPDU delay is revealed in Fig. 8 for the load of one MPDU per MAC frame. The figure shows the probability that the MPDU delay is larger than 2 ms (corresponding to two MAC frames) depending on subchannel mapping. It can be seen that the optimum subchannel mapping concerning the MPDU delay directly corresponds to to optimum in terms of PER. This is justified by the fact that at this relatively low traffic load (approximately 50 % of maximum throughput) the delay distribution is basically determined by MPDU retransmissions due to errors.

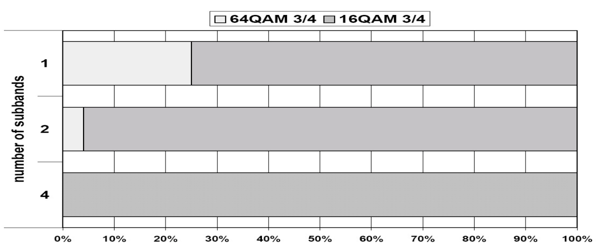

Fig. 9. PHY mode usage at system saturation (100 % resource utilization)

The mean PHY mode usage in the saturated system is shown in Fig. 9. It is only given for the first three subchannel mapping schemes. With a mapping of an OFDMA subchannel onto more than four subbands the PHY mode usage does not change any more. The results for the PHY mode usage apply for both investigated Doppler shift because it only depends on the stationary distribution of the SNR level which is the same in both cases. Corresponding to the SNR evaluation in Fig. 5, the usage of the high capacity mode 64QAM$3/4$ increases with the inter subchannel diversity due to the improved benefit of multiuser diversity.

TABLE II
MAXIMUM CONNECTION THROUGHPUT

Subbands	D_{inter}	D_{intra}	no uncertainty	100 Hz	200 Hz
1	$1/32$	$1/1$	**2.25**	2.23	1.94
2	$1/16$	$1/2$	**2.04**	2.04	1.99
4	$1/8$	$1/4$	**2.00**	2.00	2.00
⋮	⋮	⋮	⋮	⋮	⋮
32	$1/1$	$1/32$	**2.00**	2.00	2.00

The resulting maximum throughput per data connection with perfect channel knowledge and with uncertainty in the SNR estimation depending on Doppler shift is given in Table II. The results clarify the relation between both inter and intra OFDMA subchannel diversity and the system performance. With perfect channel knowledge, the maximum throughput is a monotone increasing function of the inter subchannel diversity which increases with the number of used subbands per subchannel (see Fig. 2).

Furthermore, the results show that the maximum throughput decreases for each subchannel scheme with increased uncertainty in the SNR estimation due to large Doppler shifts (up to 14 % in the case of one used subband per subchannel). An important property in this context is that the gradient of the throughput reduction also depends on the subchannel mapping. It is a monotonic decreasing function of the intra subchannel diversity which increases with the number of used subbands per subchannel.

Due to this effect, the expected throughput gains based on frequency selective scheduling cannot compensate the increased error rates when the coherence time is shorter than the duration between SNR measurement and MPDU transmission.

V. Conclusion

In this paper, a comprehensive analysis of the impact of OFDMA subchannel mapping strategies on the performance from MAC point of view has been presented. The performance evaluation has been conducted by means of stochastic event-driven simulations. The results reveal the tradeoff between throughput gains due frequency selective scheduling and accuracy in the channel state estimation due to averaging effects. The larger the Doppler shift is, which correspond to terminal velocity, the more benefits the OFDMA system from the distributed subcarrier permutations.

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