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Probability of Channel Interference for a Short Range Mobile Radio Network with Realistic Modelling of Road Traffic

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ABSTRACT: Protocol design and performance evaluation for mobile radio networks are much more difficult to solve, when a distributed instead of a centralized organization is used. We consider the problem of radio interference, resulting from the mobility of vehicles using radio equipment being able to cover only a limited range. Whenever two transmitting vehicles, originally having been sufficiently well spaced apart to communicate with their respective receivers without interference, approach each other, their transmitters interfere increasingly, if they use the same radio channel. Such interference results in an increasing bit error rate at the receivers, making finally any successful reception impossible.

Our assumption is that vehicles communicate on a channel switched basis with some of their neighboured stations and are able to decentrally and correctly decide, wether or not a channel is locally interference free to be used. This assumption is shown to be realistic by use of the Decentral Channel Access Protocol DCAP [HeWa89].

What we are interested in is the probability of channel interference for a short range mobile radio network. We consider a realistic model of road traffic, where we distinguish two cases:

* unidirectional movement of vehicles,
* bidirectional road traffic.

The performance characteristics are mathematically analysed and numeric results are presented for assumptions, representing highway, rural road and city traffic situations. It can be seen that in the one direction traffic model the resulting probability of interference is significantly smaller than that in the two opposite directions (bidirectional) traffic model.

1. INTRODUCTION

Through an electronically supported cooperation among vehicles the safety of road traffic can be considerably improved. There are considerable efforts in this area with the goal of traffic safety [ReHa88]. Consequences of this are that a large amount of information has to be exchanged efficiently between the moving vehicles and that a substantial portion of this information is only of local relevance to some more or less neighboured vehicles. Instead of vehicles communicating via their radio equipment, we shortly speak of stations. The design and performance evaluation problems in mobile radio networks are much more difficult to solve than that in stationary radio networks because

* quite different types of communication services must be implemented in the network [KrRo89] to support a driver in decision making.

* due to the limited transmission range of stations, some information might be required to be relayed several times via intermidiate stations to its destination.

* the high mobility of stations causes frequent changes of the radio connectivity between the stations, therefore the network topology is frequently changing.

We study in this paper the mobility characteristics of road traffic considering the following two facts:

1). Mobility of stations obviously plays an important role for a suitable choice of a multiple channel access method especially when communication is assumed to be channel switched, as with the protocols CSAP [MaRu88] and DCAP [HeWa89]. If the stations have a high mobility, the connectivity between stations in the system changes very frequently and the channels available must be reassigned frequently to avoid interference due to two or more transmitters occasionally using the same channel. We call this a collision of channel in this paper. Oppositely, with a lower mobility, the connectivity of stations remains quasi fixed. Without the help of a mobility model of stations it is very difficult to evaluate the performance in terms of the probability of channel interference during a given time interval, and to select some related parameter values.

2). From [BrWa89] it is known that the optimum lengths of routing update intervals and other related routing parameters directly depend upon the frequency of network topology changes. With a large update interval length the exchanged local information about network topology tends to be inactual to route data packets correctly. In contrast, with a short routing update interval, the network could be overloaded with routing related information and efficient network throughput would be substantially decreased.

Few papers were published to the authors knowledge, which deal with problems resulting of both a moving network and mobile stations. In [Ruec88] a two dimensional mobility model is introduced and mathematically analysed. The number of frames needed until a channel collision is resolved, is analysed by means of a model representing the stations mobility characteristics. One important assumption made there is, that a moving station, which is at time t in location (0,0), at time $t+\Delta t$ moves with probability $P(x,y)=1/\pi \cdot r^2$ to point (x,y), where $x^2+y^2 \le r^2$. This means that the direction of movement is homogeneously distributed in the range $(0,2\pi)$ and that the speed of all moving stations is linearly distributed in the range $(0,r/\Delta t)$, both being independent of each other, where Δt is assumed to be the frame length.

In [BrWa89] the characteristeristics of a similar model of mobility are evaluated by simulation. The performance of a decentrally operating multi-hop routing algorithm is evaluated using a piggy-backed network connectivity update procedure.

In this paper, an one-dimensional road traffic mobility model is introduced, which is more realistic and based on the following assumptions:

* in road traffic, all stations move in a line on the road with individual speeds.

* the speed of stations in the system is normally distributed, with a constant coefficient of variance [HeHo84].

Considerations of road crossings and other more realistic structures are postponed for further study. In this one-dimensional model we distinguish between two situations, namely all stations in the system are moving in two opposite directions (bidirectional model, BDM), and all stations move in the same direction (unidirectional model, UDM), representively. The letter case is applicable for the analysis of those protocols, in which transmission channels for stations are allocated according to the direction of movement [HeWa89].

From our subsequently presented ananlyse we find, that changes of network topology in the UDM appear with significantlly smaller probability than with the BDM. We further find that our BDM produces results being compareble to that of the two-dimensional mobility model, considered in [BrWa89] and [Ruec89].

In the following sections, we first introduce our linear road traffic mobility model in more detail and then analyse the characteristics of the model mathematically in section 3. In the fourth section we present and discuss the results of the analysis, and compare the results of the UDM with the BDM and with a two-dimensional mobility model of other authors. Finally, conclusions and an outlook to further studies are given.

2. VEHICLE ON ROAD MOBILITY MODEL

As mentioned above, the mobility of moving stations have impacts on the choice of the channel access method and for routing update algorithms for mobile radio networks. To analyse the road traffic mobility characteristics and their influences upon network protocols and services, we make the following assumptions:

a) All stations in the system are randomly distributed on a line, on which they move either in one direction (UDM), or in two opposite directions (BDM).

b) The number of stations in a given segment of this line is one-dimensional random variable and follows a Poisson-distribution with density λ (stations per line element).

c) Stations move independently of each other.

Assumptions b) and c) are usual for analytic models of mobile radio networks and can also be found in [TaKl84], [ZaLa88] and [Toba87].

d) The distance between two stations in the system is homogeneously distributed.

e) The line is unlimited in length.

f) All stations move along the line with random speeds of movement, which are normally distributed. Let v be the speed of a vehicle; the density of the speed distribution is then for the UDM:

$$f_{V1}(v) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{1}{2}\left(\frac{v-\xi}{\sigma}\right)^2} = Nv(\xi,\sigma) \;;$$

and for the BDM:

$$f_{V2}(v) = \frac{1/2}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{1}{2}\left(\frac{v-\xi}{\sigma}\right)^2} + \frac{1/2}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{1}{2}\left(\frac{v+\xi}{\sigma}\right)^2} = \frac{1}{2}Nv(\xi,\sigma) + \frac{1}{2}Nv(-\xi,\sigma).$$

where ξ and σ represent the mean value and standard deviation of speed.

We are interested in three representative speed distributions, which are Nv(115km/h, 20km/h), Nv(82km/h, 14km/h) and Nv(40km/h, 7km/h). The first one is the reality near speed distribution in the highway situations [HeHo84], the second and third are assumed to represent the distributions of vehicle speed of rural road and city traffic, respectively. All distributions have in commen a coefficient of variance $\xi/\sigma \approx 0.16$.

Under the above made assumptions each station can be considered to have the same statistic characteristics in the following analysis.

Fig.1 presents a road traffic mobility model, in which d is assumed to be station moving direction. If a station moves in direction d, then its speed is assumed to be positive, otherwise, negative. S is one of the stations in the system, R represents a radius around station S, which could be interpreted either as the transmit/receive radius, or the interference radius or some other radius. Ks is the range of station S, which is 2R. S' represents a station inside Ks, and S" a station outside Ks.

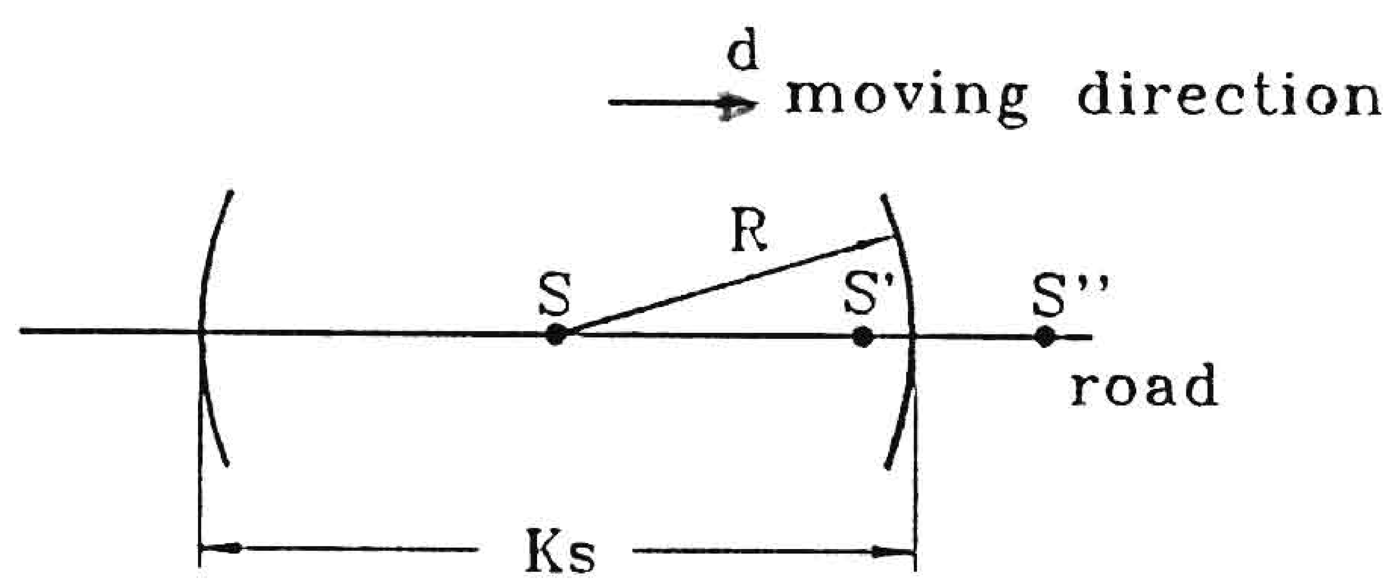


Fig.1. Linear model of road traffic mobility

One of the interesting characteristics of the mobility model is the propability p_i that exactly i (i=0,1,2,...) stations, which move at time t inside (outside) the range Ks of station S, move outside (inside) the range Ks at time t+Δt. Another interesting characteristic is the mean number of stations E(i), which leave (enter) the range Ks during the time interval Δt.

The time interval Δt could be defind e.g. by the duration of a slot in the S-Aloha protocol, the duration of a frame in a TDMA system or other time intervals like the routing update interval etc.

Under the above made assumptions the probability p_i that i stations leave and enter the range Ks during the time interval Δt is the same. Therefore we need only analyse the case that i stations leave the range Ks during Δt, and solve for the related E(i).

3. MOBILITY ANALYSIS

* Calculation of $p_{i/n}$

Pi/n defines the probability, that i stations in range Ks leave Ks during the time interval Δt, if there are n stations in the range Ks. ($i \leq n$)

The number of stations, which leave the range Ks during the time interval Δt, under the condition of n stations in this range, is binomial distributed[Ruec89], and can be expressed as follows:

$$p_{i/n} = \binom{n}{i} \cdot p^{i} \cdot (1-p)^{n-i}. \qquad (1)$$

where $p = P\{$ a station in Ks leaves Ks $\}$.
p is defined as the probability, that one station, being inside Ks at time t, moves outside Ks at least at time $t+\Delta t$.

* Calculation of p_i

The probabilty that i stations during the time interval Δt leave range Ks, in the middle of which is station S, (see Fig.1.) can be expressed in the following form according to the Bayes' Formula:

$$p_i = \sum_{n=i}^{\infty} p_{i/n} \cdot q_n \qquad (2)$$

where: q_n is the probability, that there are n stations in the range Ks.

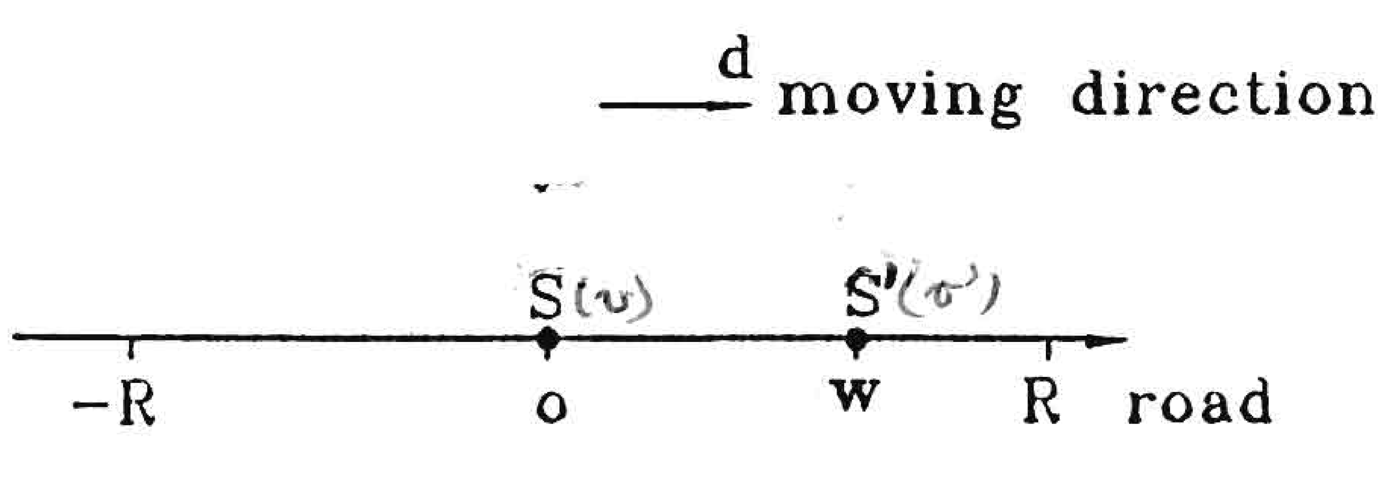
According to assumption b) we have a Poisson distributed number of stations on the line therefore :

$$\begin{aligned} p_i &= \sum_{n=i}^{\infty} p_{i/n} \cdot \frac{(\lambda \cdot 2R)^n}{n!} \cdot e^{-\lambda \cdot 2R} \\ &= \sum_{n=i}^{\infty} \binom{n}{i} \cdot p^{i} \cdot (1-p)^{n-i} \cdot \frac{(\lambda \cdot 2R)^n}{n!} \cdot e^{-\lambda \cdot 2R} \\ &= \frac{(\lambda \cdot 2R \cdot p)^i}{i!} \cdot e^{-\lambda \cdot 2R \cdot p} \end{aligned} \qquad (3)$$

* Calculation of p

In order to analyse the probability p, Let us define:

y to be the removal of station S at speed v during time interval Δt ,
z to be the removal of station S' at the speed v' during time interval Δt,
x to be the relative removal of station S' with respect to station S during time interval Δt, $x=z-y$,
and w to be the location of station S' with respect to station S at time t (see Fig.2.)



at time t

Fig.2 Calculation of $f_{X/W}(x/w)$

y, z, x and w are values of the respective random variables Y, Z, X and W and represent disdance measures. From assumption f), we know, that the density of the variables Y and Z are both normally distributed, therefore we have for the UDM:

$$f_{Y1}(y) = N_y(a,b); \qquad f_{Z1}(z) = f_{Y1}(z), \tag{4a}$$

and for the BDM:

$$f_{Y2}(y) = \frac{1}{2}N_y(a,b) + \frac{1}{2}N_y(-a,b); \qquad f_{Z2}(z) = f_{Y2}(z), \tag{4b}$$

where $a = \xi \cdot \Delta t$, $b = \sigma \cdot \Delta t$ are mean value and standard deviation respectively. We get the distribution density of the relative removal x, under the condition w:

$$\begin{aligned} f_{X/W}(x/w) &= \int_{-\infty}^{\infty} f_Y(y+w) \cdot f_Z(z)\, dz \\ &= \int_{-\infty}^{\infty} f_Y(z+w+x) \cdot f_Z(z)\, dz \end{aligned} \tag{5}$$

This is the density of the relative removal x of station S' during time interval Δt, if station S' has location w related to station S at time t.

From Eqs. (4a) and (5), we have the conditional distribution density of x for the UDM:

$$\begin{aligned} f_{X/W}(x/w) &= \int_{-\infty}^{\infty} N_{z+w-x}(a,b) \cdot N_z(a,b)\, dz \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\cdot b} \cdot e^{-\frac{1}{2}\left(\frac{z+w-x-a}{b}\right)^2} \cdot \frac{1}{\sqrt{2\pi}\cdot b} \cdot e^{-\frac{1}{2}\left(\frac{z-a}{b}\right)^2}\, dz \\ &= \frac{1}{\sqrt{2\pi}\cdot b} \cdot \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\cdot b} \cdot e^{-\frac{1}{2}\left\{\frac{\sqrt{2}[z-(a+\frac{1}{2}x-\frac{1}{2}w)]}{b}\right\}^2} \cdot e^{-\frac{1}{2}\left(\frac{x-w}{\sqrt{2}\cdot b}\right)^2}\, dz \\ &= \frac{1}{\sqrt{2\pi}\,(\sqrt{2}b)} \cdot e^{-\frac{1}{2}\left(\frac{x-w}{\sqrt{2}\,b}\right)^2} = N_x(w,\sqrt{2}\,b)\,. \end{aligned} \tag{6a}$$

Apparently the conditional density of the relative removal in the UDM is normally distributed with standard deviation $\sqrt{2}\cdot b$. This density is independent of the mean value of speed a $(=\xi\cdot\Delta t)$.

Similarly, we get the conditional density of x from Eqs. (4b) and (5) for the BDM:

$$\begin{aligned} f_{X/W}(x,w) &= \int_{-\infty}^{\infty} \left[\frac{1}{2} N_{z+w-x}(a,b) + \frac{1}{2} N_{z+w-x}(-a,b)\right] \cdot \left[\frac{1}{2} N_z(a,b) + \frac{1}{2} N_z(-a,b)\right] \cdot dz \\ &= \frac{1}{2} N_x(w,\sqrt{2}b) + \frac{1}{4} N_x(w+2a,\sqrt{2}b) + \frac{1}{4} N_x(w-2a,\sqrt{2}b)\,. \end{aligned} \tag{6b}$$

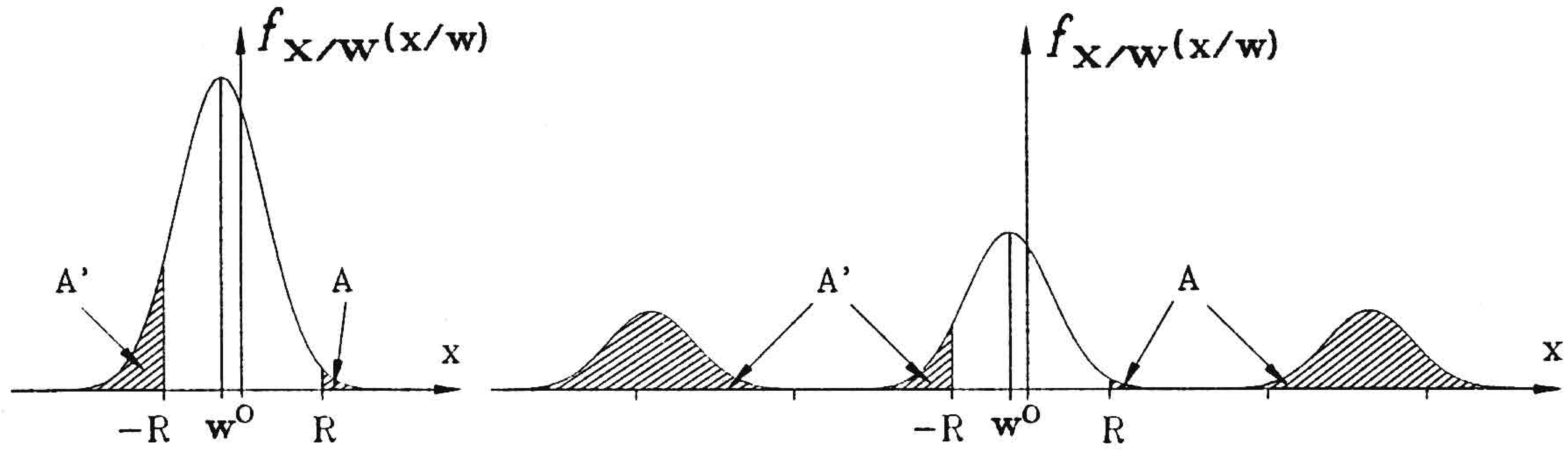
This conditional density consists of three different subnormal distributions, all having the same standard deviation $\sqrt{2}b$.

According to Figs. 1 and 2, station S' can leave the range Ks during a time interval Δt, under condition w, if and only if the relative removal x is larger than R or smaller than -R. So the conditional probability P { S' leaves Ks $w \le W \le w + \Delta w$}

can be expressed in Fig.3 as sum of the two areas of area A and A', and we have:

$$P\{ S' \text{ leaves } Ks \mid w \le W \le w+\Delta w \} = A + A'$$

$$= \int_{-\infty}^{-R} f_{X/W}(x/w)\, dx + \int_{R}^{\infty} f_{X/W}(x/w)\, dx \qquad (7)$$



a). for the UDM b). for the BDM

Fig.3. Calculation of p

The probability p is then the unconditional probability of Eq.(7):

$$p = P\{ S' \text{ leaves } Ks \}$$

$$= \sum_{w} P\{ S' \text{ leaves } Ks \mid w \le W \le w+\Delta w \} \cdot P\{ w \le W \le w+\Delta w \}$$

$$= \int_{W} P\{ S' \text{ leaves } Ks \mid w \le W \le w+\Delta w \} \cdot f_W(w)\, dw$$

$$= \int_{-R}^{R} \left[\int_{-\infty}^{-R} f_{X/W}(x/w)\, dx + \int_{R}^{\infty} f_{X/W}(x/w)\, dx \right] \cdot f_W(w) \cdot dw$$

$$= 2 \cdot \int_{-R}^{R} \left[\int_{R}^{\infty} f_{X/W}(x/w)\, dx \right] \cdot f_W(w) \cdot dw \qquad (8)$$

where $f_W(w)$ is distribution density of the distance between stations S' and S at time t, which is 1/2R according to assumption d).

So we have from Eqs.(8) and (6) the probability p, that station S' leaves the range Ks during time interval Δt for the UDM:

$$p = \frac{1}{R} \int_{-R}^{R} \left[\int_{R}^{\infty} Nx(w, \sqrt{2}b)\, dx \right] dw \qquad (9a)$$

and for the BDM:

$$p = \frac{1}{R} \int_{-R}^{R} \left\{ \int_{R}^{\infty} \left[\frac{1}{2} Nx(w, \sqrt{2}b) + \frac{1}{4} Nx(w+2a, \sqrt{2}b) + \frac{1}{4} Nx(w-2a, \sqrt{2}b) \right] dx \right\} dw \qquad (9b)$$

* Calculation of E(i)

The mean number of stations, which leave Ks during the time interval Δt can be easily determined by:

$$E(i) = \sum_{i=0}^{\infty} i \cdot p_i \tag{10}$$

4. NUMERICAL RESULTS

The results of our calculation are presented in Fig.4 to Fig.6, where the dependency of the probability p_i on the various parameters is demonstrated using Eqs. (3), (9.a) and (9.b).

In Fig.4 we see the probability pi, that i stations (i = 0,1) leave the range Ks during Δt, dependent on the station density parameter λ for the model with one moving direction (UDM). Three speed distributions representing highway, rural road and city traffic are assumed. p_2 is such small, that it can be neglected. Fig.5 presents comparable results for p_i (i=0,1,2) for the model BDM, where two opposite moving directions in the line are possible (p_3 is neglectable small). We can see intuitively from these two figures, that network topology changes during Δt in the BDM are significantly more probable than in the UDM. For example when a speed distribution N(115,20) and parameters λ = 0.04, Δt = 0.1 are assumed, the probability of any station loosing radio contact to one of its neighbours is p_1=0.0242 for the UDM and p_1=0.122 for the BDM, which is 6 times larger.

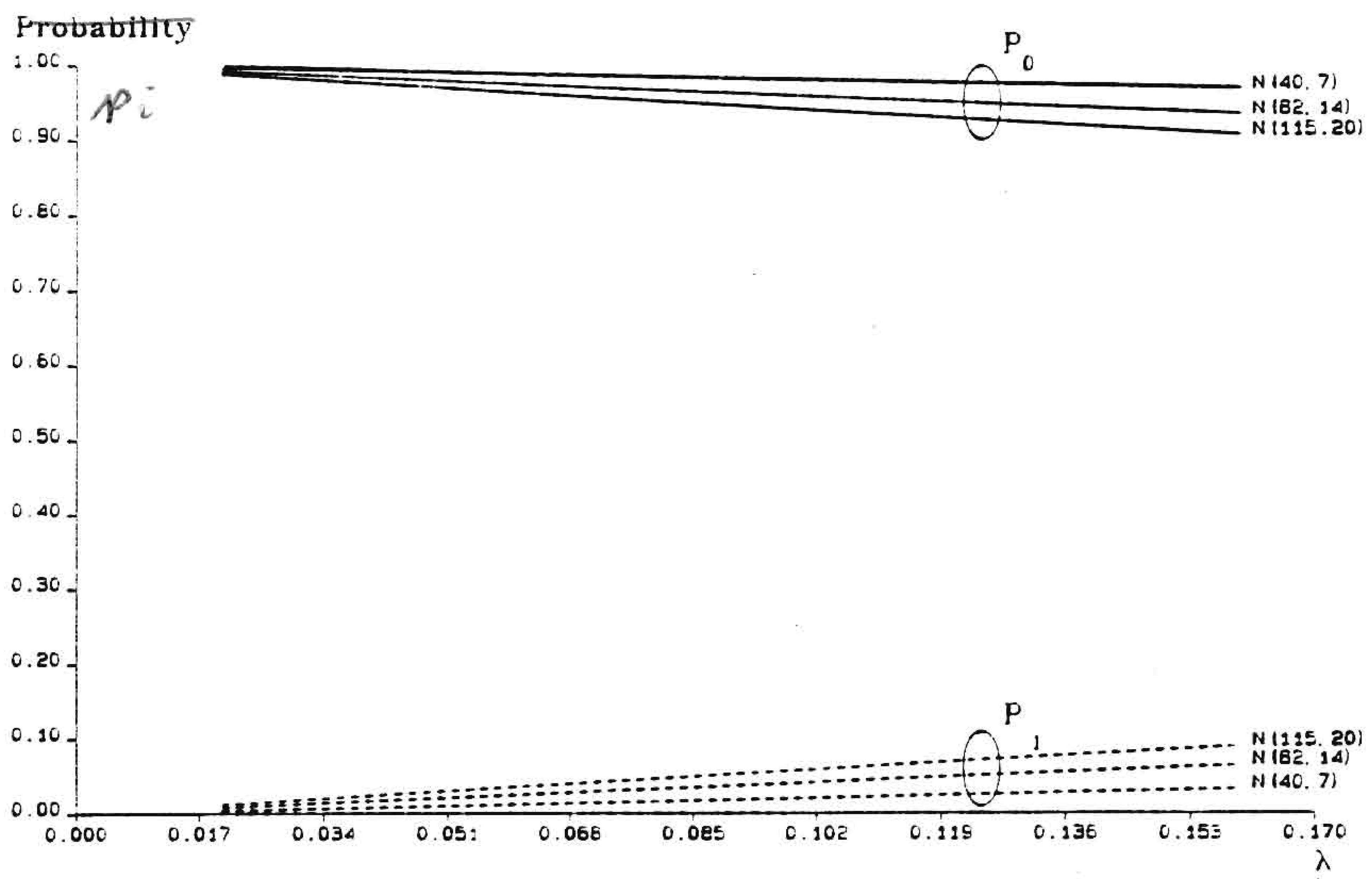


Fig.4. p_i dependent on λ for UDM(Δt=0.1sec, R=100m)

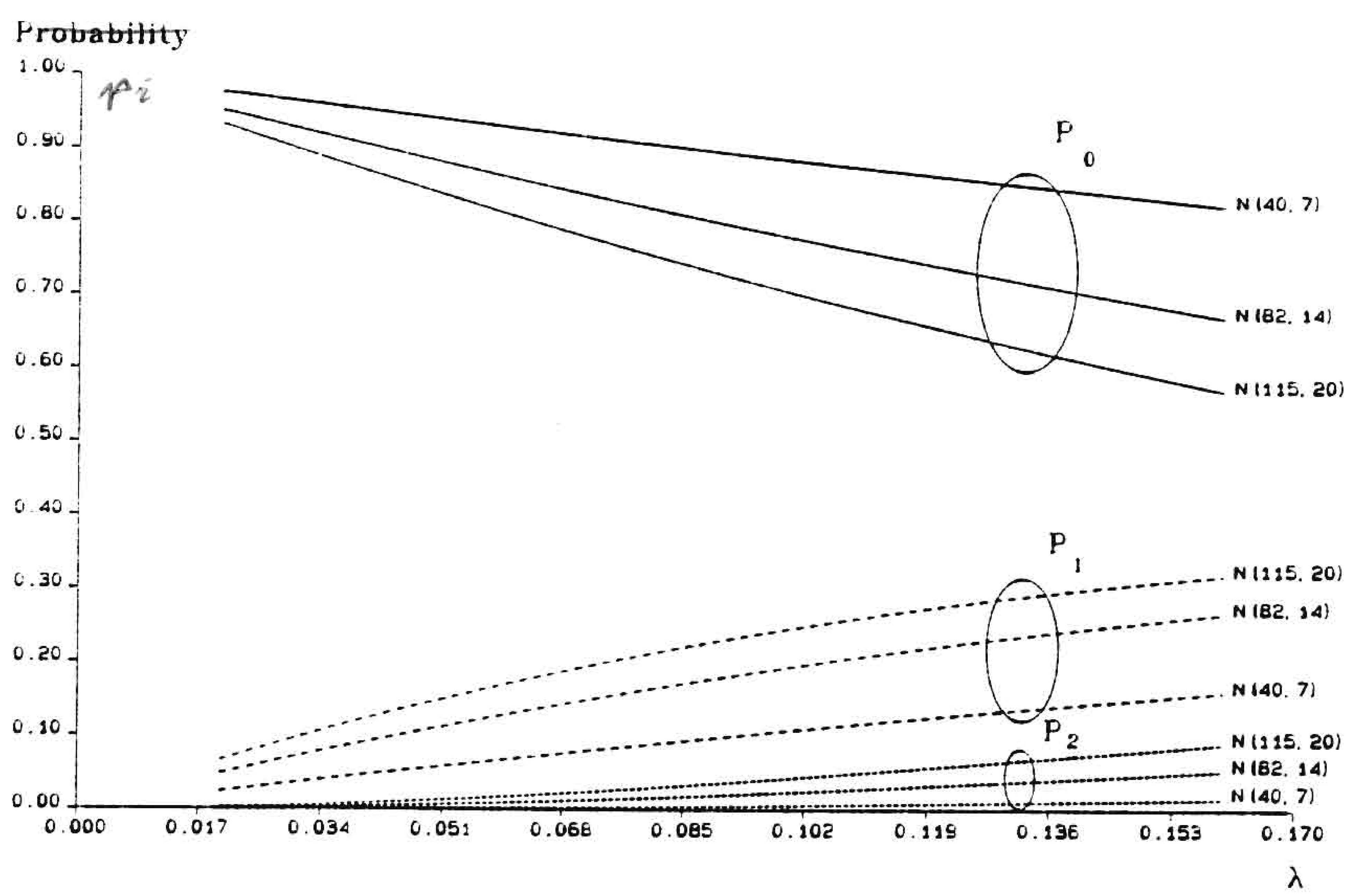


Fig.5. p_i dependent on λ for BDM (Δt=0.1sec, R=100m)

Fig.6 shows the dependency of the probability pi on the length of the time interval Δt. We find that, with increacing time interval length Δt, pi changes for the UDM more slowly than for the BDM.

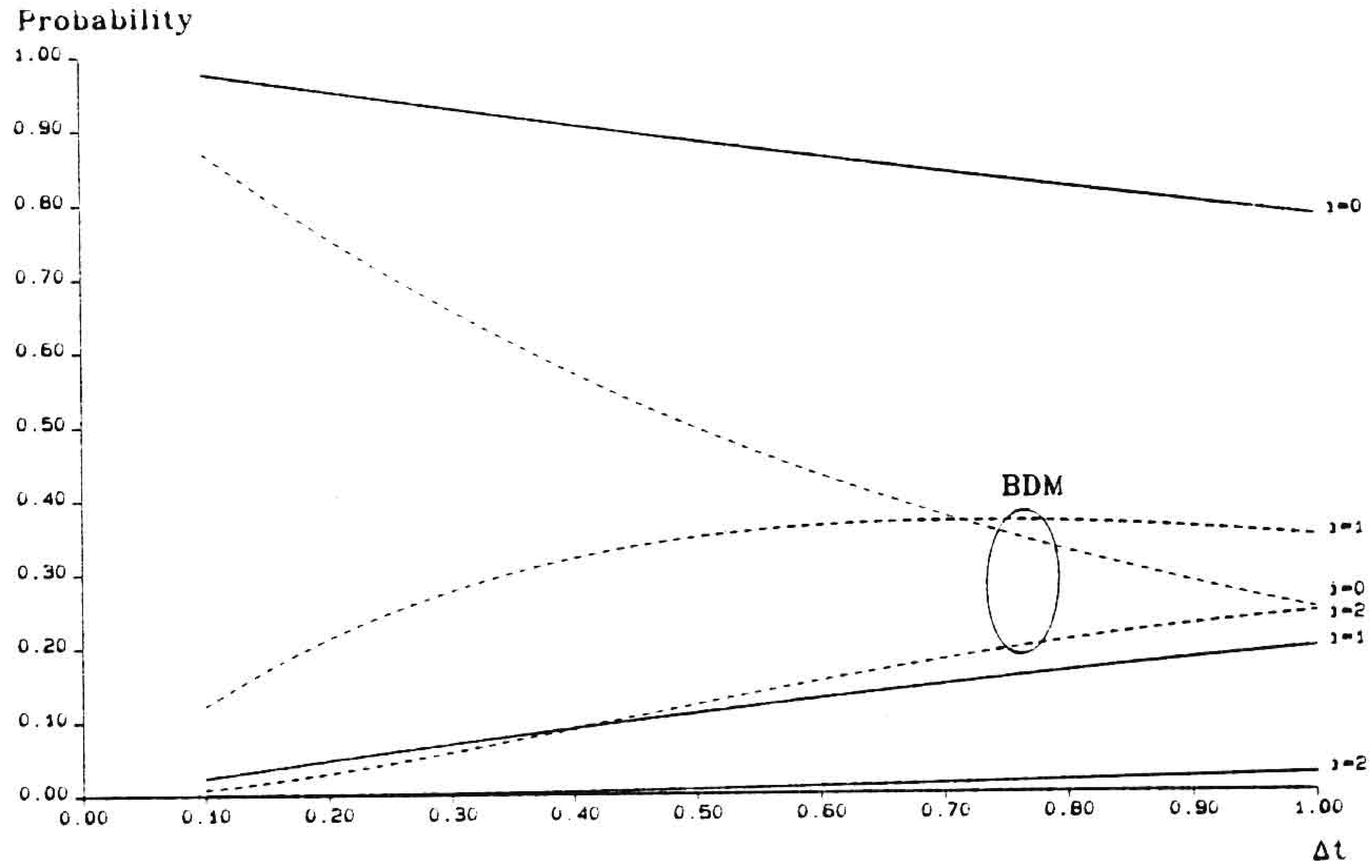


Fig.6. p_i dependent on Δt for the UDM(——) and BDM(---) for a distribution N(115,20) (λ=0.04 stations/m, R=100m)

In order to compare the results, we use Table 1, where the following parameter are assumed: mean speed ξ=180km/h, standard deviation σ = 29 km/h, radius R = 100 m, number of stations in range = λKs and time interval Δt=0.1 sec. Row #1 shows the results of the UDM, and row #2 shows that of the BDM. Row #3 represents the results of the two-dimensional model studied in [Ruec88]. We find in the table that, the topology change of the UDM is much smaller than that of the BDM, which is similar to that of the two-dimensional model (row #3).

	P0			P1			P2			P≥2		
#	λK:10	20	30	10	20	30	10	20	30	10	20	30
1	.9563	.9145	.8745	.0427	.0818	.1173	.0010	.0037	.0079	.0001	.0001	.0003
2	.7616	.5800	.4417	.2074	.3159	.3609	.0282	.0860	.1474	.0028	.0181	.0500
3	.765	.570	.430	.205	.320	.363	.026	.090	.153	.004	.020	.054

Table 1. Comparation for the distribution N(180,29), with R=100m, Δt=0.1sec.

		Δt:0.1			0.5			1.0			1.5		
#		λ:0.04	0.07	0.10	0.04	0.07	0.10	0.04	0.07	0.10	0.04	0.07	0.10
1	N(115,20)	.025	.050	.075	.125	.251	.376	.251	.501	.752	.376	.752	1.13
2	N(82,14)	.017	.035	.052	.088	.175	.263	.175	.351	.526	.263	.526	.790
3	N(40,7)	.009	.017	.026	.044	.088	.131	.088	.175	.263	.132	.263	.395
4	N(115,20)	.140	.280	.421	.702	1.40	2.10	1.40	2.76	3.92	2.10	3.92	5.41
5	N(82,14)	.100	.200	.299	.499	.999	1.50	.999	1.99	2.93	1.50	2.93	4.13
6	N(40,7)	.049	.097	.146	.244	.488	.732	.488	.976	1.46	.732	1.46	2.19

Tabel 2. Mean number E(i) of stations leaving during Δt

In Table 2 we present our results for the mean number E(i) of stations leaving a range Ks during Δt using Eqs. (3), (9a), (9b) and (10), considering R=100m as parameter. The first three row show E(i) for the UDM of three speed distributions (representing highway, rural road and city traffic situations) rows 4 to 6 represent the results for the BDM.

5. CONCLUSION

Analysis and results of a mobility model for real road traffic is presented, assuming vehicle communication via radio channels with a limited signal power. Whenever any two stations leave their respective receive ranges, a topology change of the communication network is assumed to result. We show that, when all stations in the system move in the same direction, a topology change appears with a significantly smaller probability during a fixed time interval than with a model where stations move in two opposite directions. E.g. the mean number of station which leave their respective receive range during a time interval of length 0.1 sec. is only 0.025, if stations move in the same direction, compared to 0.14 for the two opposite directions model.

Obviously this mobility model is helpful to analyse the performance and behaviour of channel access protocols and of routing table update algorithms for single- and multi-hop radio networks.

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