Local Correlation Properties of Random Sequences Generated by Queueing System *M/M/1*

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The elementary queueing system $M/M/1$ generates *correlated* random X-sequences e.g. the occupancy, the waiting or delay time, whose stationary distribution functions $F(x)$ are well known. By means of "$F(x)$ equivalent" 2-node Markov chains formulae are derived for the so-called *local* correlation coefficient $\varrho(x)$, which characterizes the correlation at any point x within the definition range of these sequences. This correlation measure is important for error statements when evaluating random sequences and is needed for the simulation as well as for the performance evaluation of queueing systems.

Lokale Korrelationseigenschaften der vom Wartesystem $M/M/1$ erzeugten Zufallssequenzen

Das elementare Wartesystem $M/M/1$ erzeugt *korrelierte* X-Zufallssequenzen, z. B. die Belegung, Warte- oder Durchlaufzeit, deren stationäre Verteilungsfunktionen $F(x)$ wohlbekannt sind. Mit Hilfe von „$F(x)$ äquivalenten" 2gliedrigen Markov-Ketten werden Formeln für den sogenannten *lokalen* Korrelationskoeffizienten $\varrho(x)$ abgeleitet, der die Korrelation an einem beliebigen Punkt x innerhalb des Definitionsbereiches dieser Sequenzen charakterisiert. Dieses Korrelationsmaß ist generell wichtig für Fehleraussagen bei der statistischen Auswertung von Zufallssequenzen und wird für die Simulation von Wartesystemen ebenso benötigt wie für deren Leistungsbewertung.

1. Introduction

In applying queueing theory to modelling and performance analysis of computer and communication systems, it is usually sufficient to make statements about the *stationary* distribution function or the *stationary* moments of the random variables of interest, such as occupancy or waiting time. But it is known that the chronological sequences of such random variables – called "random sequences" in short – show strong correlations, which might become quite significant for the evaluation of the systems and should therefore deserve more attention.

Correlation considerations are necessary when for example *error statements* about measured distribution functions or moments have to be made in real or simulated systems [5]. An extensive survey about the work up to 1974 concerning this and other aspects has been presented by Reynolds [12]. This and some newer publications [8], [17] and [18] deal mostly with the correlation coefficient [1] of order i

$$\varrho_i = \frac{\mathrm{E}\{(X_t-\bar{X})(X_{t+i}-\bar{X})\}}{\mathrm{Var}(X)}, \qquad (1)$$

which corresponds to the standard definition [9] and will be denoted in the present context as the *global* correlation coefficient representing a mean with respect to *all values* of a random variable X.

However, in order to determine the error at any point x of a measured empirical distribution function (d.f.) $F_n(x)$, a different type of first order correlation coefficient $\varrho(x)$ was defined in [15] by means of the so-called "$F(x)$-equivalent" 2-node Markov-chain [2], which corresponds to a correlation measure introduced by Blomqvist [1], [12] and is called the *local* correlation coefficient, because it characterizes the correlation in the local vicinity of any point x of the random sequence. By example of the elementary queueing system $M/M/1$, it is shown in the present paper that this local correlation coefficient $\varrho(x)$, being originally used and evaluated for simulation purposes only [16], can be described by exact formulae as part of the general queueing theory.

2. Random Sequence and Local Correlation Measure

2.1 A strict sense stationary random process produces a *continuous* random variable X resp. a random sequence

$$X=(X_1, X_2, \ldots, X_t, X_{t+1}, \ldots), \qquad (2)$$

which has the stationary d.f. and density

$$F(x) = \mathrm{Prob}\{X_t \leqq x\}; \quad f(x) = \mathrm{d}F(x)/\mathrm{d}x. \qquad (3)$$

In case of a *discrete* random sequence with $X_t \in \{0, 1, 2, \ldots\}$, the density $f(x)$ can be replaced by

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[1] In this paper it is not necessary to use the term "*auto*correlation".

[2] The 2-node Markov-chain represents the elementary generator for correlated 0/1-sequences [14] and is used instead of the "$F(x)$-equivalent" binomial generator for *uncorrelated* 0/1-sequences, which is required for the statistical determination of the empirical d.f. $F_n(x)$ of an *independent* random sequence, see e.g. [7] and [13].

its stationary probability function (p.f.)

$$P(x) = F(x) - F(x-1), \qquad x = 0, 1, \ldots \tag{4}$$

The two-dimensional d.f.

$$F(x_t, x_{t+1}) := \text{Prob}\{X_t \leqq x_t, X_{t+1} \leqq x_{t+1}\} \tag{5}$$

of two successive random variables X_t, X_{t+1} is needed for describing the stochastic properties used here, namely the *first order* correlations. In case of a continuous random sequence the associated joined density is given by

$$f(x_t, x_{t+1}) = \frac{\partial^2 F(x_t, x_{t+1})}{\partial x_t\, \partial x_{t+1}} = f(x_{t+1}|x_t)\, f(x_t), \tag{6}$$

where $f(x_t)$ is the stationary density eq. (3) and $f(x_{t+1}|x_t)$ is a conditional density. In the case of a discrete random sequence $f(x_t, x_{t+1})$ is replaced by the joint p.f.

$$P(x_t, x_{t+1}) = F(x_t, x_{t+1}) - F(x_t - 1, x_{t+1} - 1) = \\ = P(x_{t+1}|x_t)\, P(x_t). \tag{7}$$

Here $P(x_t)$ is the stationary p.f. eq. (4) and $P(x_{t+1}|x_t)$ is a conditional p.f., which in case of a homogeneous Markov-chain is represented by the well known transition probabilities.

2.2 The local correlation coefficient $\varrho(x)$ will now be defined. As shown by Fig. 1 [15], the definition range of X on the real axis is divided into two parts at point x, namely $S_0(x)$: $\{X_t \leqq x\}$ and $S_1(x)$: $\{X_t > x\}$. Now we may introduce the *local transition probabilities* $p_0(x)$ and $p_1(x)$ being defined as the conditional probabilities for the transition from the left to the right part and vice versa

$$p_0(x) := \text{Prob}\{x_{t+1} > x | x_t \leqq x\}, \quad S_0(x) \to S_1(x); \\ p_1(x) := \text{Prob}\{x_{t+1} \leqq x | x_t > x\}, \quad S_0(x) \leftarrow S_1(x). \tag{8}$$

In case of a continuous resp. discrete random sequence, the following formulae are obtained:

$$p_0(x) = \frac{\int_0^\infty \int_0^x f(x_t, x_{t+1})\, dx_t\, dx_{t+1}}{F(x)}$$

resp.

$$\frac{\sum_{x_{t+1}=x+1}^{\infty} \sum_{x_t=0}^{x} P(x_t, x_{t+1})}{F(x)};$$

$$p_1(x) = \frac{\int_0^x \int_x^\infty f(x_t, x_{t+1})\, dx_t\, dx_{t+1}}{1 - F(x)} \tag{9}$$

resp.

$$\frac{\sum_{x_{t+1}=0}^{x} \sum_{x_t=x+1}^{\infty} P(x_t, x_{t+1})}{1 - F(x)},$$

cf. eq. (6a, b) [15]. It can be shown that the double integrals resp. the double sums of the first and of the second line are equal. Now at any point x the *local correlation coefficient* $\varrho(x)$ of the sequence is identical

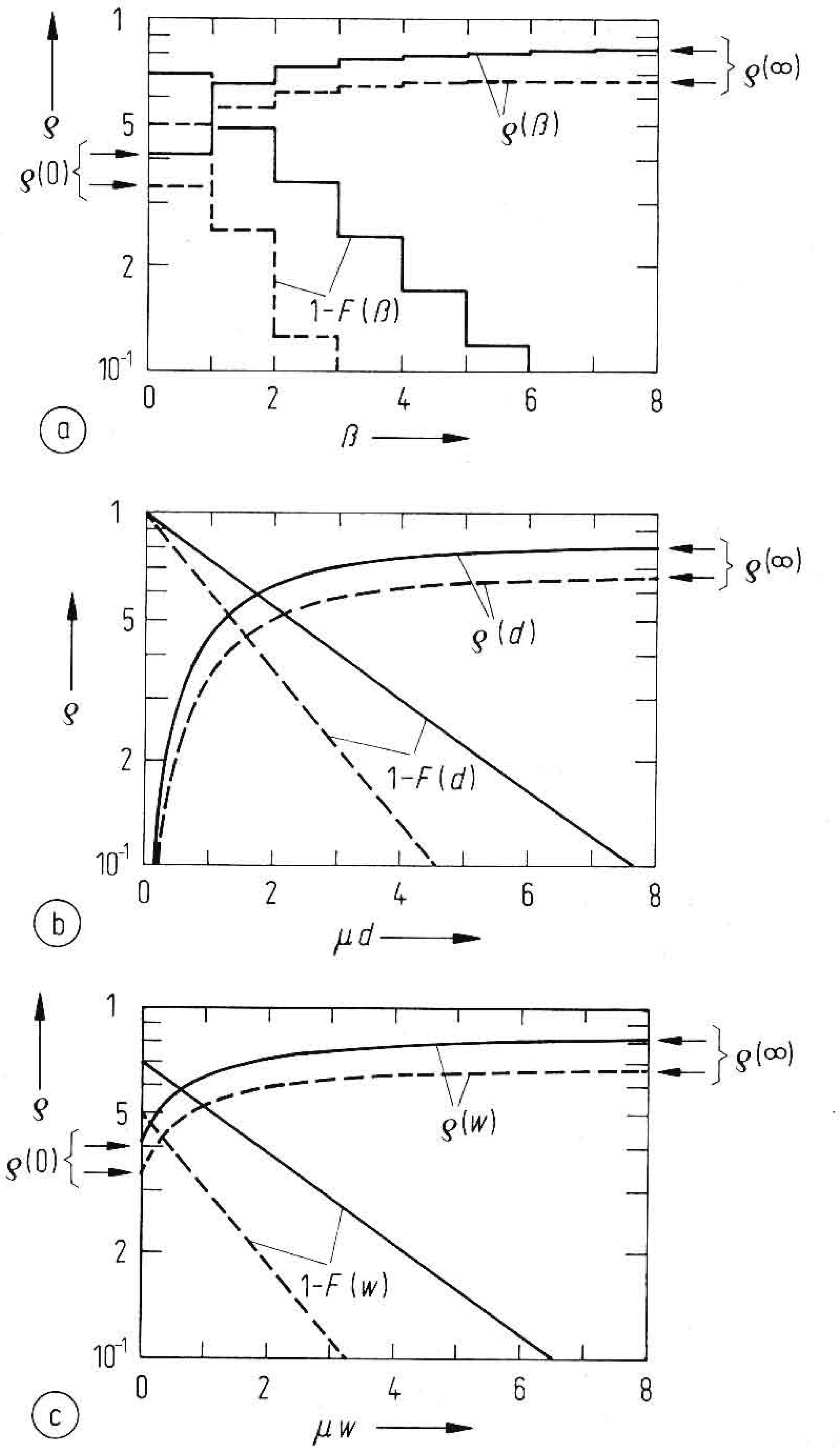

Fig. 1. Queueing system $M/M/1$: stationary compl. d.f. $1-F(x)$ and local correlation coefficient $\varrho(x)$ of different random variables. Load: $A=0.5$ dotted, $A=0.7$ solid lines. (a) Occupancy β, (b) delaytime d, (c) waiting time w.

with the correlation coefficient of an associated "$F(x)$-equivalent" 2-node Markov-chain

$$\varrho(x) = 1 - [p_0(x) = p_1(x)], \qquad -1 \leqq \varrho(x) \leqq 1, \tag{10}$$

whose stationary probability $Q(x)$ for state 0 is identical with the value of the stationary d.f. $F(x)$ eq. (3) of the considered random sequence

$$Q(x) = \frac{p_1(x)}{p_0(x) + p_1(x)} \equiv F(x), \tag{11}$$

see [14] and Section 3 [15]. From eq. (9) it is concluded for the special case "X independent" that $p_0(x) = 1 - F(x)$ and $p_1(x) = F(x)$ and that therefore $\varrho(x)$ is always equal to 0.

The coefficient $\varrho(x)$ describes the correlation of X_t, X_{t+1} at any point x and thereupon the local first order correlation properties of the random sequence X: the statement of $\varrho(x)$ characterizes the random process as a supplement to the statement of the stationary distribution function $F(x)$ [3].

[3] If we interpret x to be a signal amplitude as in statistical signal theory, then we may interpret $\varrho(x)$ to be the first order correlation coefficient *depending on the amplitude*.

3. Local Correlation Coefficients of Queueing System $M/M/1$

3.1 The Model $M/M/1$

In the queueing system $M/M/1$ considered here the time a_t between two successive arrivals of tasks N_t and N_{t+1} is exponentially distributed with the density

$$f(a) = \lambda \exp(-\lambda a). \tag{12}$$

The random sequence of the interarrival time $(a_1, a_2, \ldots, a_t, a_{t+1} \ldots)$ is independent. The system has an unlimited number of waiting positions and an exponentially distributed service time b with the density

$$f(b) = \mu \exp(-\mu b). \tag{13}$$

The random sequence of the service time $(b_1, b_2, \ldots, b_t, b_{t+1}, \ldots)$ is independent. Besides a and b are independent of each other. The service strategy is first-in first-out (FIFO). In order that the queueing system is stable, the load $A = \lambda/\mu$ must be smaller than 1.

The occupancy β_t is the number of tasks in the system *immediately after* the output of task N_{t-1}. The waiting time w_t resp. the delaytime d_t is the time between the arrival and the beginning of the service resp. the end of the output of the task N_t.

3.2 Occupancy

The occupancy β (i.e. the number of tasks in the system) is a *discrete* random variable with the definition range $\{0, 1, \ldots\}$. During the service time b_t, $k \geqq 0$ tasks can be received in the queueing system and the occupancy will change from state $\beta_t > 0$ into state $\beta_{t+1} = \beta_t + k - 1$. For $\beta_t = 0$ as a special case, the state will be $\beta_{t+1} = k$, because in case of the empty queue the first arriving task starting the service time b_t has left the system again at instant $t+1$. The number k of tasks, which arrive during service time b_t, is Poisson distributed with the p.f. $(\lambda b_t)^k \exp(-\lambda b_t)/k!$. Therefore the total probability $P_0(k)$ that k tasks have arrived during b_t is given by

$$P_0(k) = \int_0^\infty (\lambda b_t)^k \exp(-\lambda b_t)/k! \, \mu \exp(-\mu b_t) \, db_t = \frac{\mu}{\mu+\lambda}\left(\frac{\lambda}{\mu+\lambda}\right)^k = \frac{1}{1+A}\left(\frac{A}{1+A}\right)^k. \tag{14}$$

Thus we obtain the conditional p.f.[4]

$$P(\beta_{t+1}|\beta_t) = \begin{cases} P_0(\beta_{t+1}) & \text{for } \beta_t = 0; \\ P_0(\beta_{t+1} - \beta_t + 1) & \text{for } \beta_t > 0, (\beta_{t+1} - \beta_t + 1) \geqq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

In the stationary state β is geometrically distributed with the d.f. $F(\beta)$ and p.f. $P(\beta)$

$$F(\beta) = 1 - A^{\beta+1}; \quad P(\beta) = (1-A)A^\beta. \tag{16}$$

[4] Cohen has derived a similar formula for the case "immediately after an arrival" [2].

Then the joint probability according to eq. (7) is

$$P(\beta_t, \beta_{t+1}) = \begin{cases} \dfrac{(1-A)A^{\beta_{t+1}}}{(1+A)^{\beta_{t+1}+1}} & \text{for } \beta_t = 0; \\ \dfrac{(1-A)A^{\beta_{t+1}+1}}{(1+A)^{\beta_{t+1}-\beta_t+2}} & \text{for } \beta_t > 0, (\beta_{t+1} - \beta_t + 1) \geqq 0; \\ 0 & \text{otherwise,} \end{cases} \tag{17}$$

and by means of the double sum eq. (9) we obtain the local transition probabilities

$$p_0(\beta) = \frac{1-A}{1+A}\,\frac{A^{\beta+1}}{1-A^{\beta+1}}; \quad p_1(\beta) = \frac{1-A}{1+A}, \tag{18}$$

and following eq. (10) the local correlation coefficient of the occupancy β

$$\varrho(\beta) = 1 - \frac{1-A}{1+A}\,\frac{1}{1-A^{\beta+1}}; \qquad \varrho(0) = \frac{A}{1+A}; \quad \varrho(\infty) = \frac{2A}{1+A}, \tag{19}$$

see Fig. 1a.

3.3 Delaytime

The delaytime d_{t+1} of the task N_{t+1} is a *continuous* random variable, which depends on the delaytime d_t of the task foregoing N_t and on the interarrival time a_t between N_t and N_{t+1}. If a_t is greater than d_t, N_{t+1} enters in an empty queueing system. In this case, the waiting time w_{t+1} will be 0. Otherwise, N_{t+1} must wait for the service until the task N_t is finished. The waiting time w_{t+1} is equal to the delaytime d_t minus the interarrival time a_t

$$w_{t+1} = \begin{cases} 0 & \text{for } a_t \geqq d_t \\ d_t - a_t & \text{for } a_t < d_t. \end{cases} \tag{20}$$

The delaytime d_{t+1} consists of the waiting time w_{t+1} and the service time b_{t+1}

$$d_{t+1} = w_{t+1} + b_{t+1}. \tag{21}$$

By means of the Dirac-function $\delta(x)$ we can assign to eq. (20) the conditional density function

$$f(w_{t+1}|d_t, a_t) = \begin{cases} \delta(w_{t+1}) & \text{for } a_t \geqq d_t; \\ \delta[w_{t+1} - (d_t - a_t)] & \text{for } a_t < d_t. \end{cases} \tag{22}$$

Integrating over all values of a_t with "weight function" $f(a_t)$ eq. (12) a density depending on d_t only is obtained

$$f(w_{t+1}|d_t) = \int_0^\infty f(w_{t+1}|d_t, a_t) f(a_t) \, da_t = \begin{cases} \delta(w_{t+1}) \exp(-\lambda d_t) + \lambda \cdot \exp[-\lambda(d_t - w_{t+1})] & \text{for } d_t \geqq w_{t+1} \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

The density of the sum of the random variables eq. (21), which we now write in the conditional form

$d_{t+1}|d_t = w_{t+1}|d_t + b_{t+1}$, is equal to the convolution of the densities eq. (22) and $f(b_{t+1})$ eq. (13)

$$f(d_{t+1}|d_t) = f(w_{t+1}|d_t) * f(b_{t+1}) = \tag{24}$$

$$= \frac{1}{1+A} \begin{cases} \mu \exp[-\mu(d_{t+1} + A d_t)] + \\ \quad + \lambda \exp[-\mu(d_{t+1} - d_t)] & \text{for } d_{t+1} \geqq d_t; \\ \mu \exp[-\mu(d_{t+1} + A d_t)] + \\ \quad + \lambda \exp[-\mu A(d_t - d_{t+1})] & \text{for } d_{t+1} > d_t. \end{cases}$$

In the stationary state the delaytime d is described by an exponential distribution with d.f. $F(d)$ and density $f(d)$

$$F(d) = 1 - \exp[-(1-A)\mu d]; \quad f(d) = (1-A)\mu \exp[-(1-A)\mu d]. \tag{25}$$

The joint densitxy eq. (6) is then obtained

$$f(d_t, d_{t+1}) = \tag{26}$$

$$= \frac{1-A}{1+A} \begin{cases} \mu \exp(-\mu d_{t+1})[\mu \exp(-\mu d_t) + \\ \quad + \lambda \exp(\lambda d_t)] & \text{for } d_{t+1} \geqq d_t; \\ \mu \exp(-\mu d_t)[\mu \exp(-\mu d_{t+1}) + \\ \quad + \lambda \exp \lambda d_{t+1})] & \text{for } d_{t+1} < d_t. \end{cases}$$

Then the integral eq. (9) yields the local transition probabilities

$$p_0(d) = \frac{1-A}{1+A} \frac{1 - \exp[-(1+A)\mu d]}{\exp[(1-A)\mu d] - 1}; \quad p_1(d) = \frac{1-A}{1+A}\{1 - \exp[-(1+A)\mu d]\}, \tag{27}$$

and following eq. (10) we obtain the local correlation coefficient of the delaytime d

$$\varrho(d) = 1 - \frac{1-A}{1+A} \frac{1 - \exp[-(1+A)\mu d]}{1 - \exp[-(-A)\mu d]} \tag{28}$$

with the initial value $\varrho(0) = 0$ and the same end value $\varrho(\infty) = 2A/(1+A)$ as for the occupancy β, see Fig. 1 b.

3.4 *Waiting Time*

The waiting time w is treated as another example of a *continuous* variable with correlation. With $d_t = w_t + b_t$ we conclude from eq. (20)

$$w_{t+1} = \begin{cases} 0 & \text{for } a_t \geqq w_t + b_t \\ w_t + b_t - a_t & \text{for } a_t < w_t + b_t \end{cases} \tag{29}$$

and obtain the conditional density

$$f(w_{t+1}|w_t, b_t, a_t) = \begin{cases} \delta(w_{t+1}) & \text{for } a_t \geqq w_t - b_t; \\ \delta[w_{t+1} - (w_t + b_t - a_t)] & \text{for } a_t < w_t + b_t. \end{cases} \tag{30}$$

Since a_t eq. (12) and b_t eq. (13) are independent we obtain the total conditional density depending on w_t only

$$f(w_{t+1}|w_t) = \int_0^\infty \int_0^\infty f(w_{t+1}|w_t, b_t, a_t) f(b_t) f(a_t)\, db_t\, da_t = \frac{1}{1+A} \begin{cases} \delta(w_{t+1}) \exp(-\lambda w_t) + \\ \quad + \lambda \exp[-\mu(w_{t+1} - w_t)] & \text{for } w_{t+1} \geqq w_t; \\ \delta(w_{t+1}) \exp(-\lambda w_t) + \\ \quad + \lambda \exp[-\lambda(w_t - w_{t+1})] & \text{for } w_{t+1} < w_t. \end{cases}$$

The stationary d.f. $F(w)$ and density $f(w)$ of the waiting time are

$$F(w) = 1 - A \exp[-(1-A)\mu w]; \quad f(w) = (1-A)\{\delta(w) + \lambda \exp[-(1-A)\mu w]\}. \tag{32}$$

Therefore we obtain from eq. (6) the joint density

$$f(w_t, w_{t+1}) = \frac{1-A}{1+A}[\delta(w_{t+1})\delta(w_t) + \delta(\omega_{t+1})\lambda \exp(-\mu w_t) + \delta(w_t)\lambda \exp(-\mu w_{t+1})] + \frac{1-A}{1+A} \begin{cases} \lambda^2 \exp(-\mu w_{t+1} + \lambda w_t & \text{for } w_{t+1} \geqq w_t; \\ \lambda^2 \exp(-\mu w_t + \lambda w_{t+1}) & \text{for } w_{t+1} < w_t, \end{cases} \tag{33}$$

from eq. (9) the local transition probabilities

$$p_0(w) = \frac{1-A}{1+A} \frac{A \exp[-(1-A)\mu w]}{1 - A \exp[-(1-A)\mu w]}; \quad p_1(w) = \frac{1-A}{1+A}, \tag{34}$$

and thereupon the local correlation coefficient $\varrho(w)$ for the waiting time w

$$\varrho(w) = 1 - \frac{1-A}{1+A} \frac{1}{1 - A \exp[-(1-A)\mu w]} \tag{35}$$

with the same initial value $\varrho(0)$ and the same end value $\varrho(\infty)$ as for $\varrho(\beta)$ eq. (19), see Fig. 1 c.

3.5 *Evaluation of the Results*

As can be concluded from Fig. 1, the local correlation coefficients $\varrho(\beta)$, $\varrho(d)$ and $\varrho(w)$ of queueing system $M/M/1$ are positive and increase monotonously to approach the common limit $\varrho(\infty) = 2A/(1+A)$. Fig. 2 shows in case of the waiting time w the "bandwidth" of the ϱ-value as a function of the load A: the initial value $\varrho(0)$, the end value $\varrho(\infty)$ and the function $\varrho(\bar{w})$, which is obtained by introducing the mean waiting time $\mu \bar{w} = A/(1-A)$ instead of μw in eq. (35)

$$\varrho(\bar{w}) = 1 - \frac{1-A}{1+A} \frac{1}{1 - A \exp(-A)}. \tag{36}$$

As expected, the correlation increases monotonously with rising load A. The corresponding functions $\varrho(\bar{\beta})$

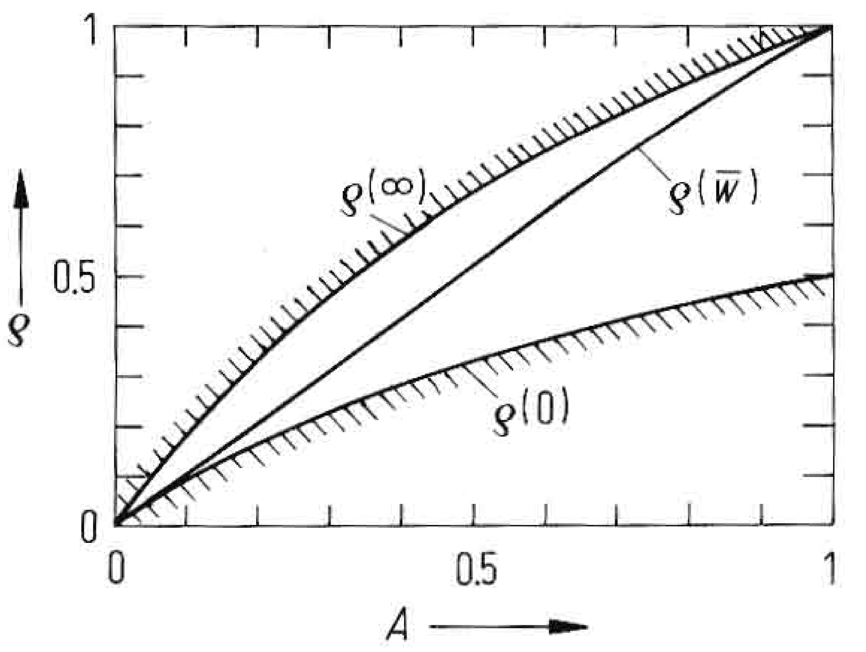

Fig. 2. Queueing system $M/M/1$: "band-width" of the local correlation coefficient $\varrho(w)$ of waiting time w as function of the load A. $\varrho(0)=A/(1+A)$, $\varrho(\infty)=2A/(1+A)$.

resp. $\varrho(\bar{d})$ can be obtained from eq. (19) and eq. (35) by substituting $\beta=\bar{\beta}=A/(1-A)$ respectively

$$\mu d=\mu\bar{d}=1/(1-A).$$

4. Conclusion

4.1 The correlation formulae $\varrho(\beta)$, $\varrho(d)$ and $\varrho(w)$ confirm the simulation results, Fig. 7a, b in [16]. As it is shown there, the function $\varrho(x)$ is required for the statistical measurement of the distribution function $F(x)$ of a correlated random sequence, in order to establish an error formula for the simulation run length control, see eq. (4) and Section 3.3 in [16].

4.2 The description of the correlation behaviour of important random variables of a queueing system such as occupancy, waiting time etc. by means of the local correlation measurement $\varrho(x)$ completes or knowledge about a queueing system and might have a practical significance under various aspects. Therefore it is interesting to investigate other queueing systems such as $M/G/1$ or the special system $M/H_2/1$ with regard to the $\varrho(x)$-functions and the effect of other service strategies instead of FIFO. New investigations [10] have shown for example that the *optimal strategy* Shortest Remaining Processing Time First (SRPT), which can be applied with considerable advantage in communication systems [6], [11], yields not only a minimal mean value and a relatively small variance of the delay time d, but also a *decorrelated* delay time, namely $\varrho(d)\approx 0$, this feature represents an additional important advantage of the SRPT-strategy.

4.3 In this paper only the local correlation coefficient of the *first order* $\varrho(x)\equiv\varrho_1(x)$ was considered. Further investigations can be devoted to the local correlation coefficients of *higher order* $\varrho_\varkappa(x)$, $\varkappa=2,3,\ldots$ as defined in in Section 6.2 [15], and to the covariance spectrums defined by Blomqvist [1].

4.4 In case of a *discrete* random variable like e.g. the occupancy β of a queueing system it might become useful for statistical purposes to describe the interaction between any *single* state β and the entirety of all other states by a "$P(\beta)$ equivalent" 2-node Markov chain and therefore by a local correlation coefficient $\varrho^*(\beta)$ being attributed to the state β under consideration. To give an example the following formula has been obtained for the occupancy β of system $M/M/1$

$$\varrho^*(\beta)=\begin{cases}\dfrac{A}{1+A} & \text{for}\quad \beta=0;\\[2ex] 1-\dfrac{1+A+A^2}{(1+A)^2\,[1-(1-A)\,A^\beta]} & \text{for}\quad \beta>0;\end{cases}$$

$$\varrho^*(\infty)=\frac{A}{(1+A)^2}\,. \qquad (37)$$

This concept can be modified to serve also in cases of a *continuous* random variable.

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References

[1] Blomqvist, N.: The covariance function of the $M/G/1$ queueing system. Skand. Aktuarietidskr. **50** (1967), 157–174.
[2] Cohen, J. W.: The single server queue. Amsterdam: North-Holland, 1969.
[3] Ding, W.; Schreiber, F.: Lokale Korrelationseigenschaften der vom Wartesystem $M/M/1$ erzeugten Zufallssequenzen. ITG-Fachbericht 107, Berlin: VDE-Verlag, 1989.
[4] Feller, W.: An introduction to probability theory and its applications, Vol. I and II. New York: John Wiley & Sons, 1970.
[5] Fishman, G. S.: Principles of discrete event simulation. New York: John Wiley & Sons, 1978.
[6] Goerg, C.: Further results on a new combined strategy based on the SRPT-principle. IEEE Trans. Comm. **37** (1989), (accepted for publication).
[7] Kendall, S. M.; Stuart, A.: The advanced theory of statistics, vol. I. London: C. Griffin & Co., 1977.
[8] Pagurek, B.; Woodside, C. M.: A recursive algorithm for computing serial correlations of times in an $M/G/1$ queue. North-Holland/TIMS Studies in the Management Sciences 7, Amsterdam: North-Holland, 1977.
[9] Papoulis, A.: Probability, random variables, and stochastic processes. Tokyo: McGraw-Hill, 1965.
[10] Perera, H. Y. R.: Beiträge zur Thorie von Wartesystemen mit den Optimalstrategien SPT und SRPT. Doctoral Thesis, Aachen University of Technology, 1989.
[11] Pham, X. H.; Schreiber, F.: The optimal strategy SRPT in Priority systems of digital communication networks. AEÜ **43** (1989), 129–134.
[12] Reynolds, J. F.: The covariance structure of queues and related processes – a survey of recent works. Adv. Appl. Prob. **7** (1975), 383–415.
[13] Schreiber, F.: The objective empirical distribution function and its error formulae. AEÜ **35** (1981), 473–480.
[14] Schreiber, F.: Measurements of the correlation coefficient of the 2-node Markov chain. AEÜ **40** (1986), 402–405.

[15] Schreiber, F.: The empirical stationary distribution function of Markovian correlated random sequences. AEÜ **41** (1987), 258–263.

[16] Schreiber, F.: Effective control of simulation runs by a new evaluation algorithm for correlated random sequences. 12th Int. Teletraffic Congress (ITC), Torino, June 1988, Paper 4.3B.1 and AEÜ **42** (1988), 347–354.

[17] Woodside, C. M.; Pagurek, B.: An algorithm for computing serial correlations of times in $GI/G/1$ queues with rational arrival processes. Management Sci. **25** (1979), 54–63.

[18] Woodside, C. M.; Pagurek, B.; Newell, G. F.: A diffusion approximation for correlation in queues. J. Appl. Prob. **17** (1980), 1033–1047.