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A Unified Correlated Input Process Model for Telecommunication Networks

Wenyue Ding

Aachen University of Technology
Kopernikusstr. 16
W-5100 Aachen
Germany

E-mail: ding@dfv.rwth-aachen.de

In this paper we introduce a correlated input process model SSMP (Special Semi-Markov Process) for telecommunication networks. Many other models, including the two-state Markov modulated Poisson process MMPP(2), separately treated in the literature are shown to be special cases. The SSMP can be applied for directly modeling discrete-time input processes which are important for the future broadband ISDN. The SSMP can be used for modeling individual processes as well as their superpositions. Based on measurements of interarrival time, fitting equations for a two-state SSMP are given and validated by simulation examples. Finally, the effects of correlations and the marginal distributions of interarrival time on queueing performance are discussed.

1 Introduction

Correlated input process models are of increasing interest for the performance evaluation of telecommunication networks. This is due to the fact that the superposition of bursty input processes, e.g. overflow processes or packetized voice sources, are often shown to be non-renewal.

Bursty properties can also be expected in the future network B-ISDN partly due to the variable bit rate video traffic. Models are provided for video sources e.g. in [9], [12]. The superposition of these video and other traffic sources such as voice, data etc. may yield a complex arrival process characterized by high peak rates, long bursts, general marginal distributions and correlations between arrivals.

Markov modulated Poisson processes, especially those having two states MMPP(2), have been successfully applied for modeling input processes, e.g. [4], [5], [7], [15], [11].

Ramaswami and Latouche provide a discrete phase type (PH) model for packet arrival from individual asynchronous input lines [14]. Fontana and Guerrero [3] proposed another correlated arrival process model called multi-mode process which is based on a *discrete-time* Markov chain and exponential distributions of the interarrival times in each state. This model aimed at characterizing the interarrival time correlations, which are discussed in the packet train model by Jain and Routhier [6].

In this paper we introduce a Special Semi-Markov Process (SSMP) for modeling input processes because of the following facts:

- Many statistical properties of input processes can be taken into account in the SSMP: e.g. arbitrary marginal distribution of interarrival times, correlations between arrivals, burstiness, etc.
- Both continuous-time and discrete-time processes can be described by SSMP. The latter is appropriate for the input processes in ATM (Asynchronous Transfer Mode) systems.
- Many other correlated input process models mentioned above can be regarded as special cases of SSMP, see section 2 and 3.
- Queues with the SSMP as input processes can be analyzed by using the methods provided in [2].

2 The SSMP model and its applicability in telecommunication networks

The SSMP is simply an extended model of the multi-mode process [3]. Instead of exponentially distributed interarrival time in each state, the SSMP allows a general distribution type. Compared with the multi-mode process and also the MMPP the advantages of our model are obvious:

1. The SSMP can fit an arbitrary marginal distribution function of interarrival time;
2. as a result of 1 the SSMP can directly describe the discrete-time processes that are useful for modeling among others the input processes of ATM systems.

An SSMP(m) has m states. Depending on state i the interarrival time has a distribution function $F_i(x)$. The state change occurs immediately after an arrival according to a Markov chain with the probability transition matrix $\mathbf{P} = (p_{ij})$. The SSMP is a special case of the semi Markov process whose interarrival distribution depends on both states i and j [13].

Let P_i be the stationary probability for state i of the Markov chain with transition matrix $\mathbf{P}$, the marginal distribution of the interarrival time is given by

$$F(x) = \sum_{i=1}^{m} P_i F_i(x). \qquad (1)$$

Let $\overline{X}_i$ be the mean interarrival time in state i and $\mathrm{Var}(X)$ the variance of the total arrival time X given by eq.(1), then the h-th order correlation coefficient between interarrival times can be obtained:

$$\rho_h = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} P_i \overline{X}_i (p_{ij}^{(h)} - P_j)\overline{X}_j}{\mathrm{Var}(X)} \qquad (2)$$

where $p_{ij}^{(h)}$ is the element of h-th order transition probability matrix defined as

$$(p_{ij}^{(h)}) = \mathbf{P}^h. \quad (3)$$

In case of $m = 2$ eq.(2) is reduced to

$$\rho_h = \frac{\kappa^h P_1 P_2 (\overline{X}_1 - \overline{X}_2)^2}{\mathrm{Var}(X)} \quad (4)$$

where κ is the first order correlation coefficient of the 2-node Markov chain given by

$$\kappa = 1 - p_{12} - p_{21} \quad (5)$$

whose stationary probabilities are given by

$$P_1 = \frac{p_{21}}{1-\kappa}, \quad P_2 = 1 - P_1. \quad (6)$$

From eq.(4) we conclude that for the conditions $\kappa > 0$ and $P_1, P_2 > 0$ and $\overline{X}_1 \neq \overline{X}_2$ the correlation coefficient ρ_h is positive and decreases geometrically with h. Another useful correlation measure will be defined in section 4.

Applying the SSMP for modeling input processes is motivated by the fact that many correlated input process models discussed in the literature can be regarded as special cases of SSMP and can be equivalently or alternatively represented by SSMP. Here are some examples:

- When the distribution function $F_i(x)$ is exponential, the SSMP degenerates into a multi-mode process [3] which can be proven to be equivalent to the well-known MMPP(2) in case of two states, see section 3. The MMPP(2) is widely used for modeling overflow processes e.g. [11] and superposition processes of packetized voice and data sources e.g. [5].
- The SSMP provides an alternative description to the discrete PH model introduced by Ramaswami and Latouche [14] for modeling individual calls in an ATM-network. Differently to a PH model e.g. Fig. 3 [14] we need no extra states for the silent period of a call, see Fig. 1. The interarrival time distribution for states $i = 2, \cdots, m$ is given by

$$F_i(x) = \begin{cases} 0, & x < d; \\ 1, & x \geq d \end{cases} \quad (7)$$

where d describes the constant interval of two successive packets (cells) during the talk spurt [2]. The interarrival time for state 1 is the sum of the silent period and d:

$$F_1(x) = \begin{cases} 0, & x < d; \\ F_{\mathrm{sil}}(x-d), & x \geq d \end{cases} \quad (8)$$

where $F_{\mathrm{sil}}(x)$ is the distribution of the silent length. The transition probability p_{1j} corresponds to the probability of the number of packets during the talk spurts.

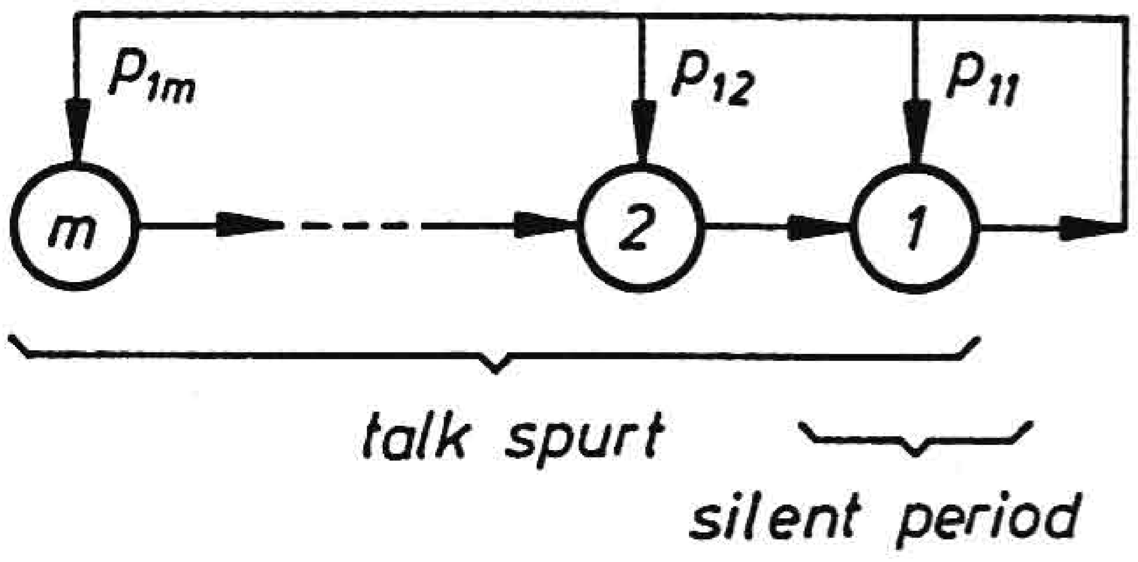

Fig. 1 The SSMP model of an individual call with talk spurt and silent period.

- SSMP is a natural candidate for modeling video sources with variable bit rate [2]. The states correspond the the levels of the arrival rates. The distribution in each state describes the slight variation of the rate.
- Batch arrivals can be easily introduced in our model by allowing $F_i(x)$ having a step at $x = 0$. In Fig. 2 it is shown a renewal process with batch arrivals. But our model is obviously not restricted to these renewal batch arrivals. The parameters are chosen as follows:

$$F_i(x) = \begin{cases} 0, & x < 0; \\ 1, & x \geq 0 \end{cases} \quad (9)$$

$$\text{for} \quad i = 2, \cdots, m$$

$F_1(x)$ corresponds to the distribution of the renewal interval, p_{1j} to the density of the number of batches.

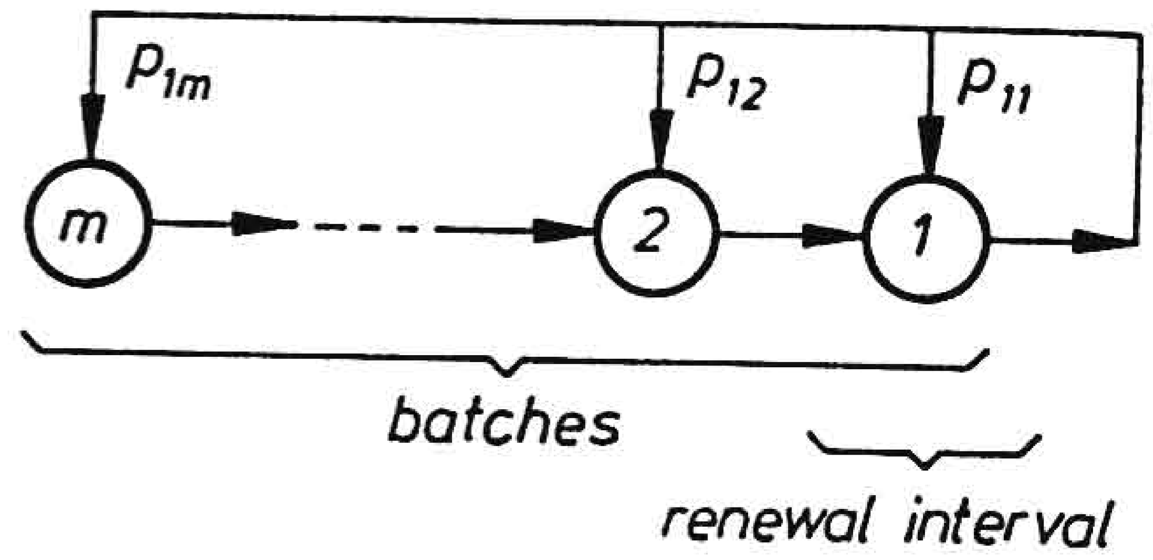

Fig. 2 SSMP as a model for renewal process with batch arrivals.

- A special case of SSMP is used for modeling a process resulting from a load distribution scheduling strategy proposed by Tran-Gia and Rathgeb [17]. The matrix P of this process has a cyclic structure, Fig.3:

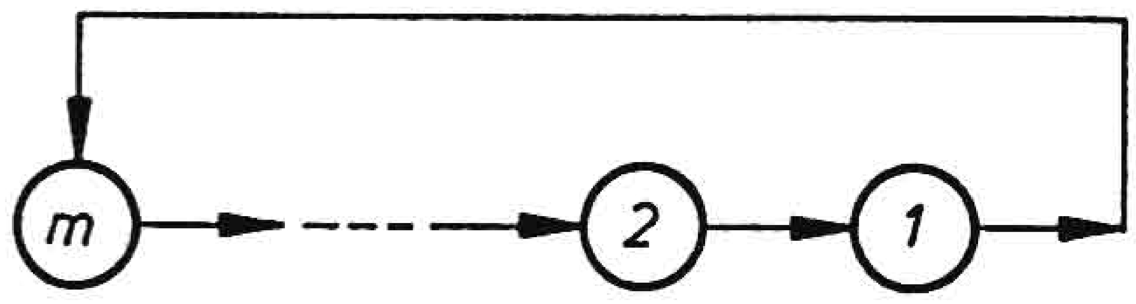

Fig. 3 A special case of SSMP used in [17].

3 Matching the MMPP(2) by a 2 state SSMP model

The MMPP(2) is applied to model input processes for several purposes, see section 1. In this section we show that the counting process of MMPP(2) can be exactly matched by the counting process of an SSMP(2) having exponential distribution in each state. A stronger result is given in [8]: The point processes generated by the MMPP(2) and its matched SSMP(2) are stochastically equivalent!

The MMPP(2) is characterized by two matrices [11]: the matrix of arrival intensities Λ and the state transition matrix of an underlying *continuous-time* Markov chain R given by

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} -r_1 & r_1 \\ r_2 & -r_2 \end{pmatrix}. \quad (10)$$

Let X_n be the time between the $(n-1)$-st and n-th arrival ($X_0 = 0$), J_n the state of the Markov chain while the n-th arrival occurs. The sequence $\{(X_n, J_n), n \geq 0\}$ is a Markov renewal sequence [13]. The transition probability matrix $\mathbf{F}(x)$ and its Laplace-Stieltjes transform $\mathbf{f}^*(s)$ are given by [10]

$$\mathbf{F}(x) = \int_0^x e^{[(\mathbf{R}-\mathbf{\Lambda})u]}\mathbf{\Lambda} du, \qquad (11)$$

$$\mathbf{f}^*(s) = \mathbf{E}\{e^{-sX}\} = (s\mathbf{I} - \mathbf{Q} + \mathbf{\Lambda})^{-1}\mathbf{\Lambda}. \qquad (12)$$

We begin to characterize the event-stationary version of MMPP(2). The stationary probability of the matrix $(p_{ij}) = \mathbf{F}(x \to \infty)$ is given by

$$\mathbf{p} = (\frac{\lambda_1 r_2}{\lambda_1 r_2 + \lambda_2 r_1}, \frac{\lambda_2 r_1}{\lambda_1 r_2 + \lambda_2 r_1}) \qquad (13)$$

The sum variable $S_n = X_1 + X_2 + \cdots + X_n$ has the distribution function

$$F_{S_n}(x) = \Pr\{S_n \le x\} \qquad (14)$$

where $F_{S_0}(x) = 1$ for $x > 0$. The density of S_n is given by

$$f_{S_n}(x) = \sum_{J_0=1}^{2} p_{J_0} \sum_{J_1=1}^{2} F'_{J_0 J_1}(x) * \cdots * \sum_{J_n=1}^{2} F'_{J_{n-1}J_n}(x) \qquad (15)$$

where $(p_1, p_2) = \mathbf{p}$ and $F_{ij}(x)$ is element of $\mathbf{F}(x)$ eq.(11). The Laplace transform can be obtained:

$$f^*_{S_n}(s) = \sum_{J_0=1}^{2} p_{J_0} \sum_{J_1=1}^{2} f^*_{J_0 J_1}(s) \cdots \sum_{J_n=1}^{2} f^*_{J_{n-1}J_n}(s) \qquad (16)$$

or in a matrix form

$$f^*_{S_n}(s) = \mathbf{p}[\mathbf{f}^*(s)]^n \mathbf{e}, \qquad (17)$$

where $\mathbf{e} = (1,1)^T$.

Let N_t be the number of arrivals during the time interval $(0,t]$. The relation between N_t and S_n is obviously

$$\Pr\{N_t < n\} = \Pr\{S_n > t\} = 1 - F_{S_n}(t) \qquad (18)$$

and

$$\begin{aligned} P_{N_t}(n) &= \Pr\{N_t = n\} = \Pr\{N_t < n+1\} - \Pr\{N_t < n\} \\ &= F_{S_n}(t) - F_{S_{n+1}}(t). \end{aligned} \qquad (19)$$

The z-Transform of $P_{N_t}(n)$ is given by

$$\begin{aligned} Q_{N_t}(z) &= \sum_{n=0}^{\infty}[F_{S_n}(t) - F_{S_{n+1}}(t)]z^n \\ &= \frac{1}{z} + \frac{z-1}{z}\sum_{n=0}^{\infty} F_{S_n}(t) z^n. \end{aligned} \qquad (20)$$

The Laplace transform of $Q_{N_t}(z)$ with respect to t is then given by

$$\begin{aligned} \Phi(s,z) &= \int_0^{\infty} Q_{N_t}(z) e^{-st} dt \\ &= \frac{1}{zs} + \frac{z-1}{zs}\sum_{n=0}^{\infty} f^*_{S_n}(s) z^n. \end{aligned} \qquad (21)$$

Inserting $f^*_{S_n}(s)$ in eq.(17), we obtain

$$\begin{aligned} \Phi(s,z) &= \frac{1}{zs} + \frac{z-1}{zs}\sum_{n=0}^{\infty} \mathbf{p}[\mathbf{f}^*(s)]^n \mathbf{e} z^n \\ &= \frac{1}{zs} + \frac{z-1}{zs}\mathbf{p}[\mathbf{I} - z\mathbf{f}^*(s)]^{-1}\mathbf{e}, \end{aligned} \qquad (22)$$

where $\mathbf{I} = \mathrm{diag}(1,\ 1)$. Finally with $\mathbf{p}$ eq.(13) and $\mathbf{f}^*(s)$ eq.(12) we have

$$\Phi(s,z) = \frac{s + \frac{bc}{d} - \frac{b}{d}(c-d)z}{s^2 + as + c - z[s(a-b) + 2c - d] + z^2(c-d)}, \qquad (23)$$

where a, b, c, d are only dependent on the parameters λ_1, λ_2, r_1, r_2 and given by

$$\begin{aligned} a &= \lambda_1 + \lambda_2 + r_1 + r_2 \\ b &= r_1 + r_2 \\ c &= \lambda_1\lambda_2 + \lambda_1 r_2 + \lambda_2 r_1 \\ d &= \lambda_1 r_2 + \lambda_2 r_1. \end{aligned} \qquad (24)$$

Now the corresponding SSMP(2) is described by the following transition matrix

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} 1-p_{12} & p_{12} \\ p_{21} & 1-p_{21} \end{pmatrix}. \qquad (25)$$

and the exponential distribution in each state

$$F_i(x) = 1 - e^{-\mu_i x}, \qquad i = 1,\ 2. \qquad (26)$$

The stationary probability vector is obtained

$$\mathbf{p} = (P_1, P_2) = (\frac{p_{21}}{1-\kappa}, \frac{p_{12}}{1-\kappa}) \qquad (27)$$

where κ is given by eq.(5). The transition distribution matrix $\mathbf{F}(x)$ and its Laplace-Stieltjes transform $\mathbf{f}^*(s)$ are given by

$$\mathbf{F}(x) = \begin{pmatrix} (1-p_{12})(1-e^{-\mu_1 x}) & p_{12}(1-e^{-\mu_1 x}) \\ p_{21}(1-e^{-\mu_2 x}) & (1-p_{21})(1-e^{-\mu_2 x}) \end{pmatrix}, \qquad (28)$$

$$\mathbf{f}^*(s) = \begin{pmatrix} (1-p_{12})\frac{\mu_1}{s+\mu_1} & p_{12}\frac{\mu_1}{s+\mu_1} \\ p_{21}\frac{\mu_2}{s+\mu_2} & (1-p_{21})\frac{\mu_2}{s+\mu_2} \end{pmatrix}. \qquad (29)$$

The formula eq. (22) is valid for all Markov renewal processes. We apply it again to obtain the Laplace transform of the z-transform of the probability $\Pr\{N_t = n\}$ in event-stationary case for this SSMP(2). From eq. (22) by inserting $\mathbf{p}$ eq.(27) and $\mathbf{f}^*(s)$ eq.(29) we have

$$\Phi'(s,z) = \frac{s + \frac{b'c'}{d'} - \frac{b'}{d'}(c'-d')z}{s^2 + a's + c' - z[s(a'-b') + 2c' - d'] + z^2(c'-d')}. \qquad (30)$$

a', b', c' and d' are for the following expressions:

$$\begin{aligned} a' &= \mu_1 + \mu_2 \\ b' &= (1-\kappa)(P_1\mu_2 + P_2\mu_1) \\ c' &= \mu_1\mu_2 \\ d' &= (1-\kappa)\mu_1\mu_2. \end{aligned} \qquad (31)$$

Comparing $\Phi'(s,z)$ eq.(30) with $\Phi(s,z)$ eq.(23), we conclude that if

$$a = a', \quad b = b', \quad c = c', \quad d = d', \qquad (32)$$

the counting processes of the MMPP(2) can be completely matched by the SSMP(2).

Finally we are able to prove that for a given parameter set λ_1, λ_2, r_1, r_2 there is a unique solution of μ_1, μ_2, P_1, P_2, $\kappa \ge 0$ and vice versa.

A stronger result is shown in [8]: If the condition (32) is fulfilled, the joint distributions of the first n $(n \ge 1)$ intervals agree. Therefore the point processes generated by the MMPP(2) and SSMP(2) whose parameters are matched by eq.(32) are in strict sense equivalent! Queueing results obtained for the one model can be transfered for the other.

4 Fitting equations of SSMP(2) by measurements of interarrival time

As mentioned in section 2 the individual cell arrival processes in ATM systems can be very closely characterized by SSMP. Unfortunately the superposition of several SSMP is not necessarily an SSMP. But an analytical description of the superposition process of individual cell arrival processes is indispensable for analytical investigations of e.g. statistical multiplexer and switching systems. For these purposes we propose an approximate method to charrcterize the superposition of cell arrivals by a SSMP(2) model. The reason why we choose the two state model is obvious:

- it is the simplest SSMP that is non-renewal;
- the two state MMPP, which is shown to be equivalent to a special case of SSMP, see section 3, is successfully used to represent the burst superposition process;
- compared with MMPP the SSMP has two further advantages: 1) fitting a general marginal distribution and 2) characterizing discrete-time processes.

Our fitting method is based on the measurement of the interarrival time sequence

$$\mathbf{X} = (X_0, X_1, \cdots, X_{n-1}, X_n, \cdots), \tag{33}$$

which can be produced both in simulated systems and in real systems. Four statistical measures should be evaluated:

1. the distribution function $F(x)$
2. the first order local correlation coefficient [1]
$$\begin{aligned}\rho_1(x) = 1 \;&-\; \Pr\{X_n > x | X_{n-1} \le x\}\\ &-\; \Pr\{X_n \le x | X_{n-1} > x\}\end{aligned}$$
3. the second order local correlation coefficient [1]
$$\begin{aligned}\rho_2(x) = 1 \;&-\; \Pr\{X_n > x | X_{n-2} \le x\}\\ &-\; \Pr\{X_n \le x | X_{n-2} > x\}\end{aligned}$$
4. the joint distribution function
$$h(x) = \Pr\{X_n \le x,\; X_{n-1} \le x, X_{n-2} \le x\}$$

The measured process should be approximated by an SSMP(2) whose theoretical statistical measures mentioned above can be given by

$$F(x) = P_1F_1(x) + P_2F_2(x) \tag{34}$$

$$\rho_1(x) = \frac{\kappa P_1P_2[F_1(x) - F_2(x)]^2}{F(x)[1 - F(x)]} \tag{35}$$

$$\rho_2(x) = \frac{\kappa^2 P_1P_2[F_1(x) - F_2(x)]^2}{F(x)[1 - F(x)]} \tag{36}$$

$$\begin{aligned}h(x) = \;& \kappa P_1P_2(F_1(x) - F_2(x))^2\cdot\\ & [\kappa(P_1F_2(x) + P_2F_1(x)) + 2F(x)] + F^3(x)\end{aligned} \tag{37}$$

which can be used as fitting equations for the parameters and functions of the SSMP(2):

$$\kappa,\; P_1,\; P_2,\; F_1(x),\; F_2(x).$$

4.1 Estimation of κ

For each x we can get an estimation of κ from eq.(35) and (36)

$$\kappa(x) = \frac{\rho_2(x)}{\rho_1(x)}. \tag{38}$$

For the reason that the measurements of functions $\rho_1(x)$ and $\rho_2(x)$ are less accurate for small $F(x)$ and $1 - F(x)$ we use a weight function for the final estimation of κ:

$$w(x) = F(x)[1 - F(x)]. \tag{39}$$

The final estimation will be made by weighting the function $\kappa(x)$:

$$\tilde{\kappa} = \frac{\int_0^\infty w(x)\kappa(x)dx}{\int_0^\infty w(x)dx} \tag{40}$$

In discrete-time case the integrals lead to sums.

4.2 Estimation of P_1 and P_2

We introduce two auxiliary functions $A(x)$ and $B(x)$ defined by

$$A(x) = \frac{\rho_1(x)F(x)[1 - F(x)]}{\tilde{\kappa}} \tag{41}$$

$$B(x) = \frac{\frac{h(x) - F^3(x)}{\tilde{\kappa}\,A(x)} - 2F(x)}{\tilde{\kappa}}. \tag{42}$$

The temporary estimations of P_1 and P_2 are dependent on x. From eq.(34) and (37) we obtain for each x

$$[P_1(x) - P_2(x)][F_1(x) - F_2(x)] = F(x) - B(x) \tag{43}$$

Suppose $P_1(x) \ge P_2(x)$, we get a condition for $F_1(x)$ and $F_2(x)$ that will be used in the sequel:

$$\begin{aligned}F_1(x) \ge F_2(x) \quad &\text{if} \quad F(x) \ge B(x);\\ F_1(x) < F_2(x) \quad &\text{if} \quad F(x) < B(x).\end{aligned} \tag{44}$$

Finally, from eq.(35) and (43) under the condition $P_1(x) + P_2(x) = 1$ we have

$$P_1(x) = \frac{1}{2}[1 + \frac{|F(x) - B(x)|}{\sqrt{4A(x) + (F(x) - B(x))^2}}] \tag{45}$$

$$P_2(x) = 1 - P_1(x) \tag{46}$$

The final estimation $\tilde{P}_1$ and $\tilde{P}_2$ can be made by using the weight function eq.(39), analogous to eq.(40).

4.3 Estimation of $F_1(x)$ and $F_2(x)$

The last step will be the estimation of $F_1(x)$ and $F_2(x)$, but it is the least problematic one due to the known values of $\tilde{\kappa}$, $\tilde{P}_1$ and $\tilde{P}_2$. From eq.(34) and (35) we obtain

$$\tilde{F}_1(x) = F(x) \pm \sqrt{\frac{\tilde{P}_2}{\tilde{P}_1}A(x)}; \tag{47}$$

$$\tilde{F}_2(x) = F(x) \mp \sqrt{\frac{\tilde{P}_1}{\tilde{P}_2}A(x)}, \tag{48}$$

The +/−signs are decided by the condition given in eq.(44). Finally the transition matrix (p_{ij}) can be obtained:

$$\begin{aligned}p_{12} &= \tilde{P}_1(1 - \tilde{\kappa}), \quad p_{11} = 1 - p_{12}\\ p_{21} &= \tilde{P}_2(1 - \tilde{\kappa}), \quad p_{22} = 1 - p_{21}\end{aligned} \tag{49}$$

The fitting equations presented above are exact in mathematical sense, if the original process is an SSMP(2) and the mesurements of $F(x)$, $\rho_1(x)$, $\rho_2(x)$ and $h(x)$ are exact. However one could get some statistical problems by implementing this procedure, due partly to the finite length of the sequence measured and partly to the fact that the original process is not exactly an SSMP(2).

The measurements of the tails need a considerablly large number of events. We suggest the LRE (Limited Relative Error) algorithm which guarantees that the relative error of the complementary distribution function $1 - F(x)$ at any point x does not exceed a prescribed value [16].

Some values of $\kappa(x)$ may be out of the definition range, but it is not a problem one should worry about. We do not need to take these values into consideration, since at least one value of $\kappa(x)$ is necessary for the whole procedure. Similar heuristic could be applied for $P_1(x)$ and $P_2(x)$. For $\tilde{F}_1(x)$ and $\tilde{F}_2(x)$ there is another method necessary: if $\tilde{F}_i(x) < 0 (i = 1 \text{ or } 2)$, we replace it with 0; if $\tilde{F}_i(x) > 1 (i = 1 \text{ or } 2)$, we replace it with 1; finally, if the estimated density of $\tilde{F}_i(x)$ is negative ($i = 1$ or 2), we replace the density with 0.

The SSMP(2) can cover a wide range of superposition processes, but no doubt it is restricted to some classes. We leave it for our future work to find out which type of superpositions can be fitted by SSMP(2). An extensive simulation study is indispensable. For superpositions which are unable to be fitted by SSMP(2), a more state SSMP should be suggested.

For many cases the functions $F_1(X)$ and $F_2(x)$ can be described (approximately) by some simple analytical functions having only few parameters. Efficient fitting algorithms should be developed for these cases. However we emphasize that the SSMP renders possible to take into account the arbitrary distributions and therefore it is a powerful model.

5 Waiting time distributions: an example

Now we apply the fitting algorithm for the superposition of discrete-time SSMP's. To illustrate the effects of the marginal distribution and correlations on queueing systems, we approximate the original superposition process by four processes: GEO, GI, SSMP(GEO,GEO), SSMP(GI,GI). The notation SSMP(A,B) stands for an SSMP(2), whose interarrival time distribution has type A and B in state 1 and 2, respectively. The GEO process has a geometric distribution function

$$F(x) = 1 - q^{k+1} \quad \text{for} \quad k \leq x < k+1 \qquad (50)$$

with the mean $\overline{X} = \frac{q}{1-q}$. The geometric decay parameter q can be estimated by the measured mean interarrival time $\overline{X}^*$:

$$\tilde{q} = \frac{\overline{X}^*}{1+\overline{X}^*}. \qquad (51)$$

The GI type has a general discrete distribution which can be fitted by taking only the measured marginal distribution into account. SSMP(GEO,GEO) can be considered to be a discrete analogy of MMPP(2), which is widely applied in modeling the superposition processes in communication networks. The parameters of the SSMP(GEO,GEO) can be fitted by first using the fitting method of a SSMP(2), see section 4 and calculating the means $\overline{X}_1$, $\overline{X}_2$ for state 1 and 2 and then computing q_1, q_2 by using eq.(51). The SSMP(GI,GI) is a better fitting process than SSMP(GEO,GEO), since both correlation and the general distribution of the original process can be fitted.

The waiting time distribution of the discrete SSMP/G/1 queue is analyzed in [2]. SSMP/G/1 obviously includes the very special case GI/D/1 and SSMP(2)/D/1, in which the arrival process has only one or two states, respectively. The service time has a deterministic distribution. The choice of this service time distribution is encouraged by the fact that the packets (cells) in ATM systems have a constant length of 53 octets, which leads to a constant service time for each cell.

The simulated superposition process in our example includes the following individual processes:

- one source of SSMP(GEO,GEO) with the geometric decay parameters $q_1 = 0.8$ and $q_2 = 0.5$. The parameters of the matrix **P** are given by $p_{12} = 0.18$ and $p_{21} = 0.12$ which lead to $P_1 = 0.4$ and $\kappa = 0.7$ and
- two sources of GEO with the common geometric decay parameters $q = 0.925$.

Fig.4 shows the complementary waiting time distributions of the $\cdot$/D/1 queue. The service time has a constant normalized length of 1. The input processes are the simulated superposition process described above and its four approximations: GEO, GI, SSMP(GEO,GEO) and SSMP(GI,GI). The waiting time distributions of the queues with approximated input processes are computed by the algorithm described in [2]. The waiting time distribution of the queue with the original superposition process is evaluated from the simulation by a procedure based on the LRE algorithm [16]. The prescribed maximal relative error of the compl. waiting time distribution $G(x)$ is given by 5% for $G(x) \geq 10^{-3}$.

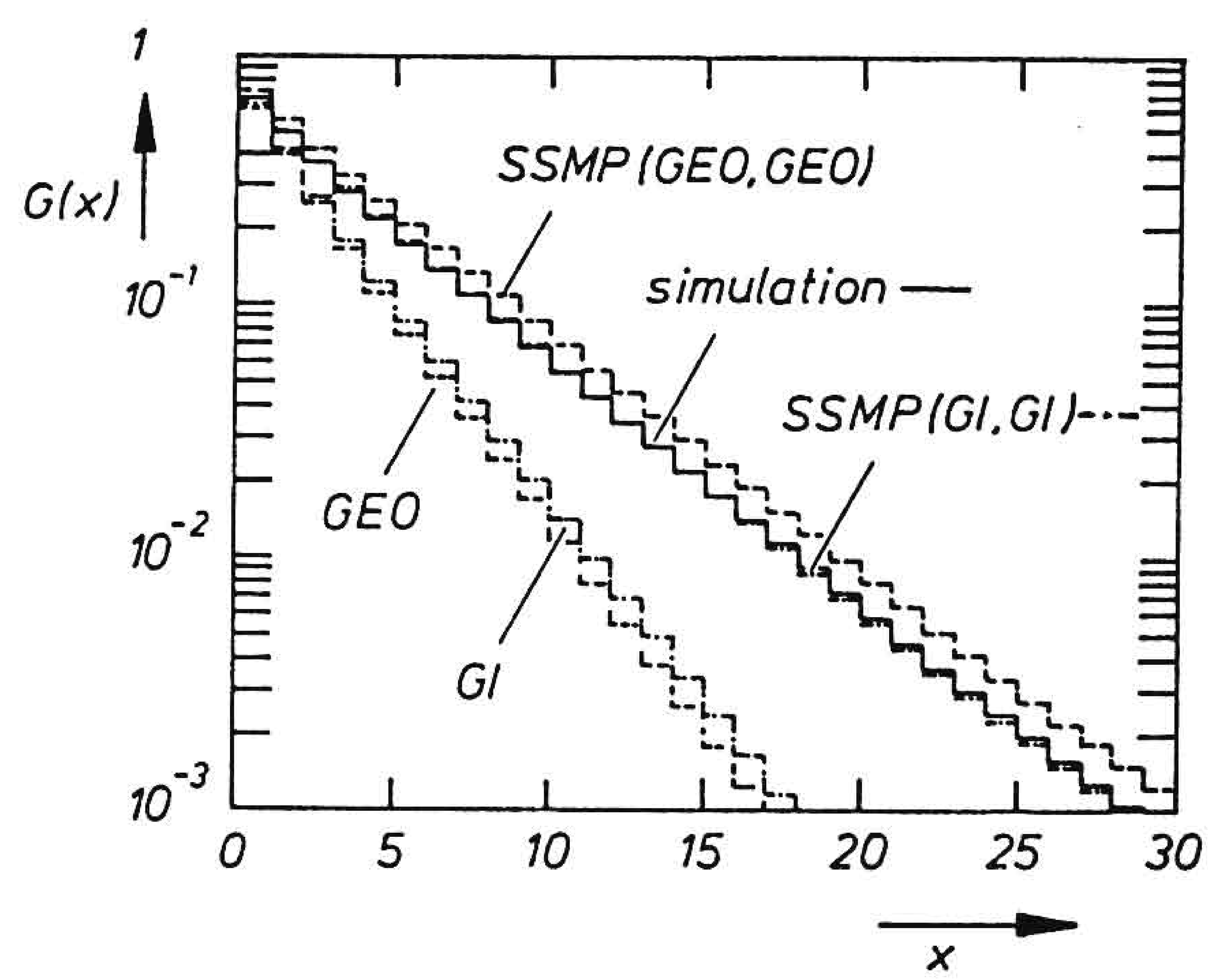

Fig. 4 Compl. waiting time distributions of the $\cdot$/D/1 queue with the simulated superposition input process on the one side and approximated input processes on the other side: GEO, GI, SSMP(GEO,GEO) and SSMP(GI,GI).

From Fig. 4 we can conclude that the SSMP(GI,GI) is a very accurate approximation of the simulated superposition process in this example, while the GEO and GI approximations underestimate and the SSMP(GEO,GEO) approximation slightly overestimates.

6 Conclusion

In this paper we have introduce a special semi-Markov process (SSMP) for modeling correlated input processes in telecommunication networks. The applicability of the SSMP model in real systems is shown by the fact that many correlated input process models including the well-known MMPP(2), which is successfully applied for modeling the superposition of burst processes, can be considered to be special cases of SSMP. Compared with MMPP(2), the SSMP has two further advantages:

1. a general marginal distribution can be taken into account;
2. as a results of 1 it can be used for directly modeling discrete-time processes, which is important for performance evaluation a.o. in B-ISDN based on the ATM principle.

The SSMP can be applied for modeling both the individual cell arrival processes and their superposition. A fitting algorithm for the SSMP(2) having two states is given, based on the statistical measurement of the interarrival times. This fitting algorithm is validated by simulation examples. The algorithm is also appropriate for measurements of real systems.

To illustrate some effects of the distribution and correlation of input processes on queueing systems we have approximated a superposition process by four processes:

- GEO: only mean interarrival time of the measurement is considered;
- GI: only marginal distribution of the measurement is considered;
- SSMP(GEO,GEO): correlation and mean in each state are considered;
- SSMP(GI,GI): both distribution and correlation are considered.

The waiting time distributions of $\cdot/D/1$ queues with the approximated input processes are analyzed by using the method presented in [2]. Compared with simulation results, both distributions and correlations of interarrival times cannot be neglected in general. Therefore geometric approximations as well as renewal approximations of interarrival times should be very carefully examined for certain cases.

Our future work will include characterizing the superposition input processes for ATM systems, eventually by applying the fitting method described in section 4. For this purpose simulation studies as well as measurements of real systems will be important.

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