Performance Analysis of the Random Access Protocol in TETRAPOL Trunked Radio Networks

Dirk Kuypers, Peter Sievering
Communication Networks
Aachen University of Technology, D–52074 Aachen, Germany
{dk,psi}@comnets.rwth-aachen.de

Abstract

This paper provides a performance evaluation of the TETRAPOL random access protocol. The results are based on a Markovian model which is also presented. The Markovian model is used to derive quality of service parameters, e.g. mean throughput, waiting probability, and mean random access delay. The analytical results are compared to results based on stochastic simulation. Our prototypic implementation of the TETRAPOL protocol stack uses the *Specification and Description Language* (SDL). The simulation results correspond to the analytical results with only small differences. Furthermore, these results are compared to the maximum number of users which can be served by TETRA systems. In case of one *Control Channel* (CCH) and a typical traffic load, up to approx. 300 mobile stations can be served by one TETRAPOL base station.

1 Introduction

New trunked mobile radio systems are about to be implemented in European countries. Both standards, *Terrestrial Trunked Radio* (TETRA) developed by the *European Telecommunications Standards Institute* (ETSI) and TETRAPOL developed by Matra Nortel Communications, provide pure digital information technology for the transmission of speech and data and have been chosen as platforms for the operation of nation-wide trunked radio networks in Europe. As TETRA and TETRAPOL are competitors in the market of *Professional Mobile Radio* (PMR), it is essential to compare the performance of both system types. When planning and dimensioning a cellular network, the number of *Base Stations* (BS) and the cell sizes depend on the number of users, which can be served by a base station. As the frequency spectrum is limited in bandwidth, the number of *Control Channels* (CCH) per base station, which are necessary to exchange control information with a given number of *Mobile Stations* (MS) is another key factor regarding the network planning.

The throughput and the connection set-up time are important *Quality of Service* (QoS) parameters and mainly determined by the waiting time for a free server and the access delay according to the random access protocol, which is used to transmit capacity requests.

Starting with an outline of the TETRAPOL protocol stack at the air interface in Section 2, we then describe in Section 3 the TETRAPOL random access protocol. In Section 4

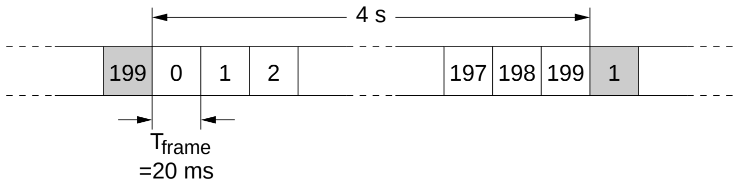


Figure 1: Structure of the TETRAPOL superframe [19]

we present our analytical model based on a Markov chain and derive quality of service parameters, e.g., mean throughput, waiting probability, and mean access delay. After this, in Section 5 these parameters are compared to results based on stochastic simulations. Finally, the number of users, which can be served by a base station, is compared to the performance of a TETRA system in Section 6.

2 The TETRAPOL public available specification

The TETRAPOL trunked radio system has been developed by Matra Nortel Communications and has already found acceptance in a number of European countries for public safety forces (police, fire brigades, customs etc.). Since the TETRAPOL specification is public available (*Public Available Specification*, PAS) now, more companies have decided to develop TETRAPOL products.

In this section we give a short outline of the TETRAPOL system and its protocol stack at the air interface.

2.1 Technical data

The TETRAPOL system can be used as a local or a multicell network. It can be operated in the frequency bands between 70 MHz and 520 MHz. A TETRAPOL base station can handle up to 24 radio channels.

The TETRAPOL channel access is based on *Frequency Division Multiple Access* with 12.5 kHz channel spacing. The gross modulation bit rate is 8 kbit/s using binary *Gaussian Minimum Shift Keying* modulation. Each radio channel provides one bi-directional control or traffic channel and carries a set of logical channels. At least one control channel, which is called *Master Control Channel* (MCCH), is known to all mobile stations in a radio cell. Multiple *Extended Control Channels* (ECCH) can be defined in a radio cell to extend the signalling capacity. Both, MCCH and ECCH can not be used as traffic channels. A traffic channel can either be used to transmit circuit-switched or packet-switched data or circuit-switched speech frames.

As shown in Figure 1, a radio channel's structure in time is described by the subsequent repetition of so-called superframes, which last 4 s and consist of two-hundred 160-bit frames, numbered from 0 to 199. Each logical channel uses a part of the superframe structure [19]. The length of a single frame is $T_{frame} = 20\,\mathrm{ms}$.

2.2 Services

The TETRAPOL system provides packet-data services, circuit-switched data, and speech services. The packet-oriented services differentiate between the following types of connections:

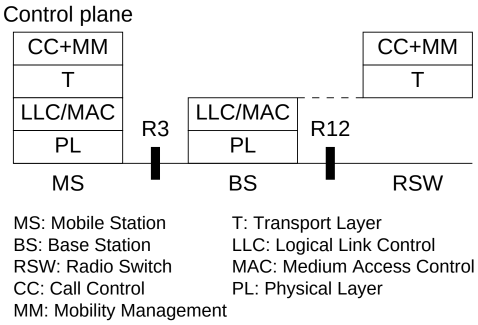


Figure 2: Definition of the reference point R_3

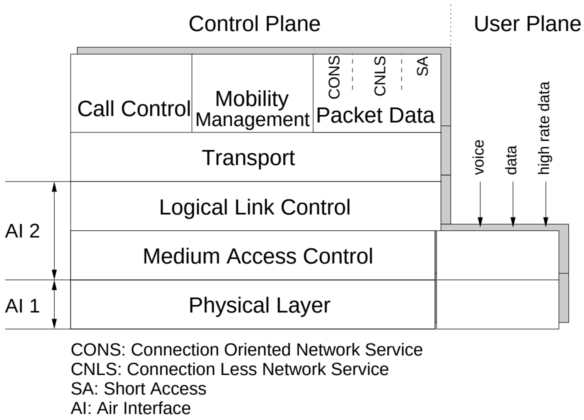


Figure 3: The TETRAPOL air interfaces protocol stack

- Connection-oriented packet-data transmission based on ITU-T X.25.
- Connectionless packet-data transmission for non-acknowledged point-to-multipoint services.

Circuit-switched speech can be transmitted protected via teleservices. The teleservices for speech transmission offer six types of connection:

Individual call Point-to-point connection between calling and called subscribers.

Conference call Point-to-multipoint connection between calling and up to four called subscribers in half-duplex mode.

Group call Point-to-multipoint connection between calling subscriber and a group called through a common group number. The call is set-up quickly because no confirmation is required. The communication takes place in half-duplex mode.

Direct call Point-to-point connection between two mobile devices with no use of the infrastructure.

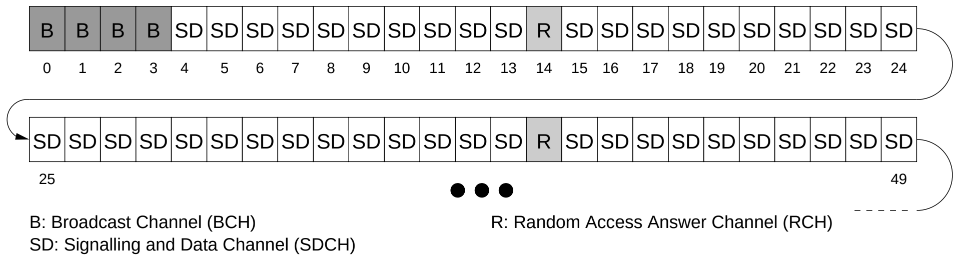
Broadcast call Point-to-multipoint connection in which the called group dialled through a broadcast number can only hear the calling subscriber.

Emergency call With activating an emergency call, the terminal transmits a status message. The dispatcher can either activate an open channel or set-up a connection with high priority to the calling subscriber.

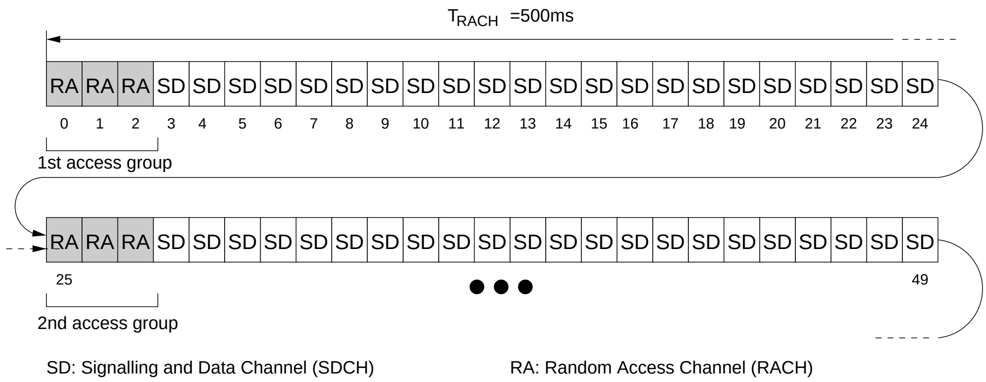
2.3 Protocol stack

The reference point between radio terminal and radio base station is denoted as R_3 (see Figure 2). The protocol stack at reference point R_3 comprises three layers (see Figure 3): the *Physical Layer* (PL); the data link layer, which is divided into *Medium Access Control* (MAC) and *Logical Link Control* (LLC); and the *Transport* (T) layer, which is divided into several sublayers and offers management services to base and mobile stations. The MAC layer is based on two protocol stacks: the *User Plane*, which is responsible for information transport, and the *Control Plane* for signalling.

Each time, an application transaction is initiated or re-established, the mobile station uses the random access protocol, which is described in the next section.



(a) *Downlink*



(b) *Uplink*

Figure 4: Logical channel organization

3 TETRAPOL random access protocol

On the uplink control channel, the frames $(i \cdot 25)$, $(1 + i \cdot 25)$, and $(2 + i \cdot 25)$ with $i = 0, ...7$, form eight groups of *Random Access Channels* (RACH) in a superframe with three RACH frames each (see Figure 4(b)). In the following we denote these groups of RACH frames as *access groups*. Thus, the duration between two consecutive access groups is $T_{RACH} = 500\,\mathrm{ms}$.

The random access channel frames are used by the mobile stations to initiate or re-establish a data link layer connection. Therefore, a mobile station chooses one of the three random access frames in the next access group by evaluating the last four bits of its link level terminal identifier [18]. Hence, all mobile stations in a radio cell are uniformly distributed on the three random access channel frames of an access group. Within the chosen random access channel frame, the mobile station transmits the 14 most significant bits of its link level terminal identifier.

The base station acknowledges the successful receptions of random access channel frames in an access group using the next *Random Access Answer Channel* (RCH). The RCH frame is also used to indicate collisions, if two or more mobile stations access the same random access channel frame at the same time, or non receptions, if non of the random access channel frames sent by mobile stations are received by the base station. The RCH frames are transmitted every 500 ms in frames $(14 + i \cdot 25), i = 0, ...7$. The waiting time for an acknowledgment (positive or negative) on the RCH channel after an access on the random access channel is 13–15 frames, i.e. 260–300 ms (see Figure 4).

The resolution of collisions is performed in the mobile station. Figure 5 depicts a mobile

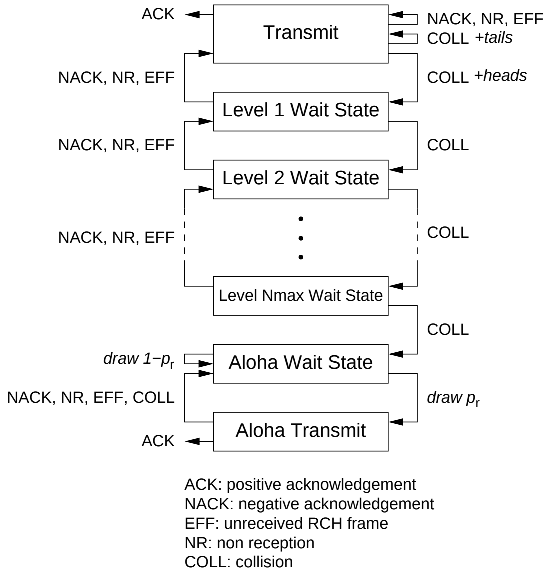


Figure 5: Random access retransmission algorithm [18]

station's state machine during random access, but not the access itself. The states of the collision resolution algorithm and the events that cause state transitions are shown. If a mobile station needs to initiate or re-establish a data link layer connection, e.g., due to a packet data transmission request or a speech connection set-up request, it enters the *Transmit* state and uses the next random access channel frame. If a collision (COLL) occurs, the mobile station changes to the *Level 1 Wait State* with probability $\frac{1}{2}$ depending on the result of a random heads and tails decision. If residing in wait states, a mobile station listens to the random access answer channel without transmitting on the random access channel. Subsequent collisions of the access of other mobile stations effect this waiting mobile station to change to a higher wait state level. The reception of random access answer channel frames indicating non receptions (NR), acknowledgments for other mobile stations (NACK), or un-received random access channel frames (EFF) makes the waiting mobile station reduce its wait state level. In the *Transmit* state it again accesses the next random access channel frame.

The events NR, NACK and EFF arise from transmission failures due to fading, co-channel interference or shadowing on uplink (NR or NACK) or downlink (EFF).

If the maximum wait state level N_{max} is exceeded, the mobile station changes to a *Slotted-ALOHA* (S-ALOHA) access procedure. The S-ALOHA access probability is constant in time and denoted as p_r. With a successful access (ACK) the random access procedure is finished. The state, that is taken then, is hidden in Figure 5.

From a mobile station's point of view, that resides in a transmit state (states *Transmit* or *Aloha Transmit*), a collision occurs, if one or more other mobile stations access the same next random access channel frame. In comparison, if that mobile station resides in a wait state (i.e. not in transmit states), a collision occurs, if at least two other mobile stations access the next random access channel frame.

A typical value for the S-ALOHA access probability is $p_r = 0.05$ and a typical value for the number of wait state levels is $N_{max} = 8$ [18].

4 Analytical model

The analytical model is described in three steps. At first, a Markov chain is presented, which models the mobile stations collision resolution algorithm. Then, the corresponding system of equations is solved for typical algorithm parameters. Finally, formulae are derived regarding complementary distribution functions of the random access delay and mean throughput.

4.1 Markovian model

Due to the fact that a mobile station uses only one random access channel frame of an access group (see Section 3), in the following only one random access channel frame per access group is considered.

Let N_{MS} be the number of identical mobile stations generating transmission requests according to a Poisson distribution with mean arrival rate λ. New connection set-up requests are transmitted in the next random access channel frame. In case of stationary equilibrium, the total send rate of all mobile stations equals to $N_{MS} \cdot \lambda$.

A mobile station's state of the random access protocol can be described by the number of requests to send $s(t)$ and the wait state level $w(t)$. The random variables $s(t)$ and $w(t)$ have discrete values and can change each random access answer channel frame. Thus, the two-dimensional process $(s(t), w(t))$ can be described by a time-homogeneous *Discrete Time Markov Chain* (DTMC) with a distance of $T_{RACH} = 500\,\text{ms}$ between the time points of observation. We assume, that each mobile station has not more than one request to transmit at the same time, i.e. $s(t) \in \{0, 1\}$. Then, the probability for a new request arrival between two time points of observation is $0 \leq \lambda T_{RACH} \leq 1$.

State $(0, 0)$ describes an idle mobile station. This state is hidden in Figure 5. State $(1, N_{max} + 1)$ denotes the *Aloha Wait State*, whereas state $(1, N_{max} + 2)$ marks the *Aloha Transmit* state.

If the loss of frames due to fading, co-channel interference and shadowing effects is not taken into account, the probabilities for the events NR, NACK, and EFF are zero for mobile stations that are in a transmit state. Furthermore, the collision probability p_{c1} is the probability that at least another mobile station accesses the same random access channel frame.

Otherwise, if the mobile station is in a wait state, the collision probability p_{c2} is the probability that two or more other mobile stations transmit on the random access channel at the same time. If ideal transmission conditions are assumed, the probabilities for the events ACK and EFF are zero. Then, the two-dimensional process $(s(t), w(t))$ is shown in Figure 6.

The underlying Markov chain has the transition matrix $\mathbf{P}$, where $p(j_1, k_1|j_0, k_0)$ denotes the transition probability from state (j_0, k_0) to state (j_1, k_1):

$$\mathbf{P} = \begin{pmatrix} p(0,0|0,0) & \cdots & p(1, N_{max}+2|0,0) \\ \vdots & \ddots & \vdots \\ p(0,0|1,N_{max}+2) & \cdots & p(1,N_{max}+2|1,N_{max}+2) \end{pmatrix} \tag{1}$$

$\mathbf{P}$ is a stochastic matrix, because all its entries satisfy $0 \leq p(j_1, k_1|j_0, k_0) \leq 1$ and $\sum_{(j_1,k_1)} p(j_1, k_1|j_0, k_0) = 1 \quad \forall (j_0, k_0)$. As each random access starts in state $(0, 0)$, the state probability vector at time n is [7, 5]:

$$\underline{\pi}(n) = \{p_n(0,0), \ldots p_n(1, N_{max}+2)\} = \underline{\pi}(0)\mathbf{P}^n \quad \text{with} \quad \underline{\pi}(0) = \{1, 0, \ldots 0\} \tag{2}$$

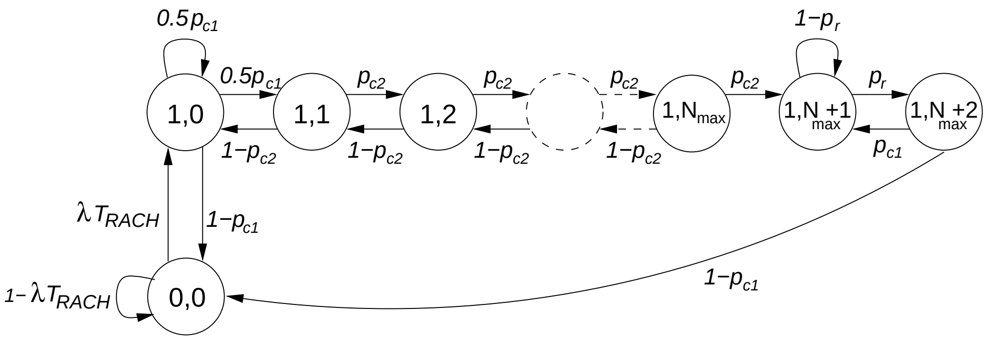


Figure 6: State transition diagram of the Discrete Time Markov Chain

$\underline{\pi}(n)$ is a stochastic vector with $0 \leq p_n(j,k) \leq 1$ and $\sum_{(j,k)} p_n(j,k) = 1 \quad \forall n$.

For the case of stationary equilibrium, we calculate the probability that a mobile station sends on the random access channel, i.e. we calculate the probabilities for states $(1,0)$ and $(1, N_{max}+2)$. Thus, the stationary access probability τ is:

$$\tau = \lim_{n\to\infty} (p_n(1,0) + p_n(1, N_{max}+2)) = \lim_{n\to\infty} (\underline{\pi}(0)\mathbf{P}^n) \cdot \{0,1,0,\ldots 0,1\} \tag{3}$$

The steady-state distribution vector $\underline{v} = \lim_{n\to\infty}(\underline{\pi}(0)\mathbf{P}^n)$ can be determined by solving the system of linear equations $\underline{v} = \underline{v}\mathbf{P}$ [5]. The access probability τ includes the first access and retransmissions and depends on the collision probabilities p_{c1} and p_{c2}, see Figure 7. To determine τ, p_{c1}, and p_{c2}, two more equations are needed.

If a mobile station is in state $(1,0)$ or $(1, N_{max}+2)$, the collision probability p_{c1} is the probability that one or more other mobile stations access the next random access channel frame. With N_{MS} mobile stations we find with a product-form approach:

$$p_{c1} = \sum_{i=1}^{N_{MS}-1} \binom{N_{MS}-1}{i} \cdot \tau^i \cdot (1-\tau)^{N_{MS}-1-i} = 1 - (1-\tau)^{N_{MS}-1} \tag{4}$$

The collision probability p_{c2} for a mobile station, that does not access the next RACH frame, i.e. that is in a wait state, is the probability that at least two of the other N_{MS} mobile stations access the same next random access channel frame:

$$p_{c2} = \sum_{i=2}^{N_{MS}-1} \binom{N_{MS}-1}{i} \cdot \tau^i \cdot (1-\tau)^{N_{MS}-1-i} = p_{c1} - (N_{MS}-1) \cdot \tau \cdot (1-\tau)^{N_{MS}-2} \tag{5}$$

Equations 3, 4, and 5 are non-linear and no explicit solution is known for these general parameters. Furthermore, the solution depends on the number of wait state levels N_{max}. According to [18], the typical number for the number of wait state levels is $N_{max} = 8$. Figure 8 depicts the results, that have been calculated numerically using Maple [21].

4.2 Complementary distribution function of the random access delay

The random access delay is defined as the duration between the arrival of a new transmission request and the reception of an acknowledgment of the successful transmission on the random access channel. To determine the complementary distribution function of the random access delay, we first calculate the probability for returning to state $(0,0)$ for the first time. Each state transition is done after an additional delay of T_{RACH}. By applying

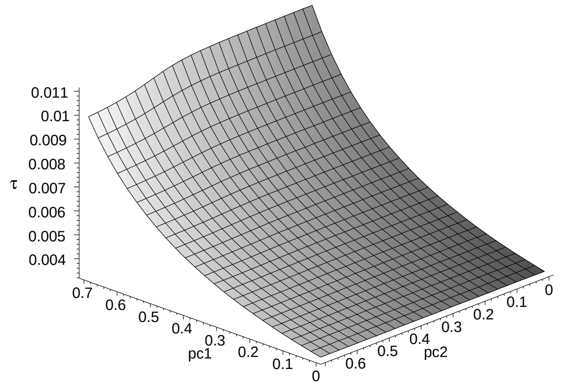


Figure 7: Access probability τ versus collision probabilities for $\lambda T_{RACH} = 0.00335$

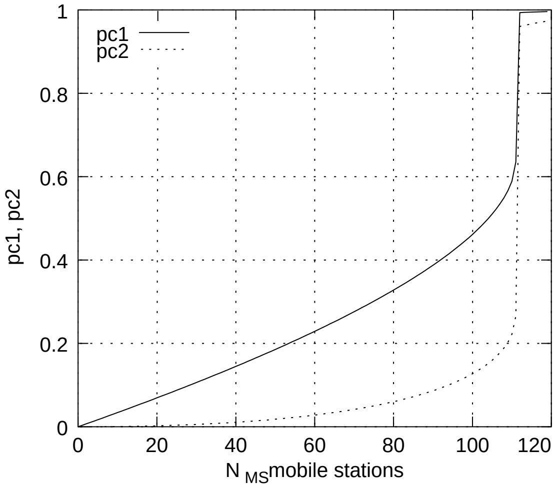


Figure 8: Probabilities p_{c1} and p_{c2} for $\lambda T_{RACH} = 0.00335$ and $N_{MS} \in [1, 120]$

the absorbing Markov chain theory to the Markov chain in Figure 6, we change the process $(s(t), w(t))$ by making state $(0, 0)$ an absorbing state. As proven in [7], the behaviour of this process is exactly the same as the one of the original process before reaching state $(0, 0)$ for the first time.

State $(0, 0)$ is made absorbing with $p(0, 0|0, 0) = 1$ and $p(1, 0|0, 0) = 0$. The resulting transition matrix is of canonical form:

$$\mathbf{P}' = \left(\begin{array}{c|c} 1 & \mathbf{0} \\ \hline \mathbf{R} & \mathbf{Q} \end{array} \right)$$

where $\mathbf{R}$ and $\mathbf{Q}$ are the unchanged $1 \times (N_{max} + 2)$ and $(N_{max} + 2) \times (N_{max} + 2)$ sub-matrixes of the original process (see Equation 1). The starting vector is $\underline{\pi}'(0) = \{0, 1, 0, \ldots 0\}$. Hence, the probability that the process starting in transient state $(1, 0)$ ends up in absorbing state $(0, 0)$ at time n is (see Equation 2):

$$P_{Acc}(T = n) = \underline{\pi}'(0)\mathbf{P}'^{n} \cdot \{1, 0, \ldots 0\}$$

The expected value for the random access duration is $E[T_{Acc}] = \sum_{i=1}^{\infty} i \cdot P_{Acc}(T = i)$, and its time-discrete *Complementary Distribution Function* (CDF) can now be determined by:

$$P_{Acc}(T > n) = 1 - P_{Acc}(T = j)$$

As described in Section 3, the random access answer sent by the base station is received 13–15 frames after an access on the random access channel. As the N_{MS} identical mobile stations are uniformly distributed on the three random access channel frames in an access group, the waiting time between an access on the random access channel and the corresponding acknowledgment is uniformly distributed on [260 ms,300 ms] with CDF $P_{Ack}(T > t)$.

The CDF of the random access delay also takes into account the on $[0, T_{RACH}]$ uniformly distributed duration between the arrival of a new transmission request and the begin of the next random access channel frame with CDF $P_{Arr}(T > t)$. Thus, the continuous CDF of the random access delay is:

$$P(T > t) = P_{Arr}(T > t) \otimes P_{Acc}(T > n) \otimes P_{Ack}(T > t) \tag{6}$$

4.3 Mean random access delay

The expected value for the waiting time between the arrival of a new transmission request and the next *Random Access Channel* (RACH) frame is $E[T_{Arr}] = 0.5 \cdot T_{RACH} = 250\,\mathrm{ms}$. $E[T_{Ack}] = 280\,\mathrm{ms}$ is the expected value for the waiting time between the access on the RACH and the reception of an acknowledgment on the random access answer channel. With Equation 6, the expected value of the random access delay equals:

$$E[T_W] = \bar{T}_W = E[T_{Arr}] + E[T_{Acc}] + E[T_{Ack}] = E[T_{Acc}] + 530\,\mathrm{ms}$$

with $E[T_{Acc}] = \sum_{i=1}^{\infty} i \cdot P_{Acc}(T = i)$.

4.4 Mean throughput

Three states of an RACH frame can be differentiated by a base station:

empty An RACH frame is called empty, if no access takes place during this frame. The probability of an empty RACH frame is $p_e = (1-\tau)^{N_{MS}}$.

success A transmission on an RACH frame is successful, if exactly one mobile station transmits. The probability is $p_s = N_{MS} \cdot \tau \cdot (1-\tau)^{N_{MS}-1}$.

collision If two or more mobile stations access an RACH frame at the same time, a collision occurs. The collision probability is $p_c = 1 - p_e - p_s$.

The mean throughput per RACH frame can be noted as $\bar{S} = p_s$ because a successful access on the RACH has the probability p_s. Figure 9 shows that the maximum throughput $\bar{S}_{max} \approx e^{-1}$ is reached for $N_{MS} = 112$ ($\lambda T_{RACH} = 0.00335$). A stable operation in this scenario is only possible up to this maximum number of mobile stations. The figures also show, that the collision probabilities p_{c1} and p_{c2} increase rapidly, if the random access channel is slightly overloaded. The maximum number of mobile stations is limited to $N_{MS} = 186$, if $\lambda \cdot T_{RACH} = 0.002$ is assumed (see Figure 9). Due to the three RACH frames per access group, a total number of $3 \cdot N_{MS} = 336$ ($\lambda \cdot T_{RACH} = 0.00335$), resp. $3 \cdot N_{MS} = 558$ ($\lambda \cdot T_{RACH} = 0.002$) mobile stations can be served by one base station, if only the throughput of the RACH is taken into account.

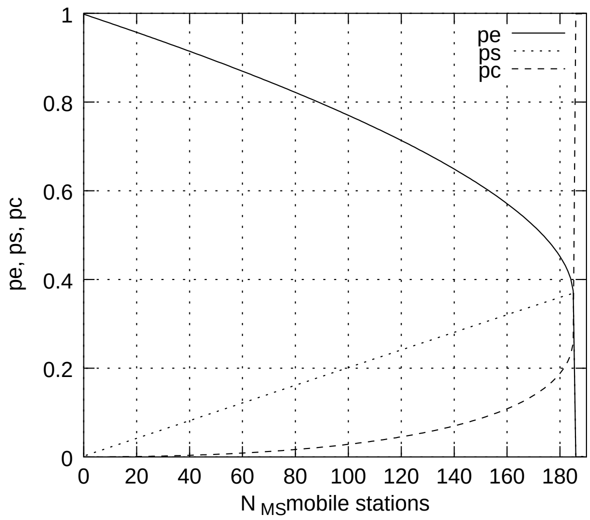
In Figure 9, p_s seems to be linear dependent on N_{MS}. This is because $\tau \ll 1 \Rightarrow (1-\tau) \approx 1 \Rightarrow p_s \approx N_{MS} \cdot \tau$.

5 Performance evaluation by simulation

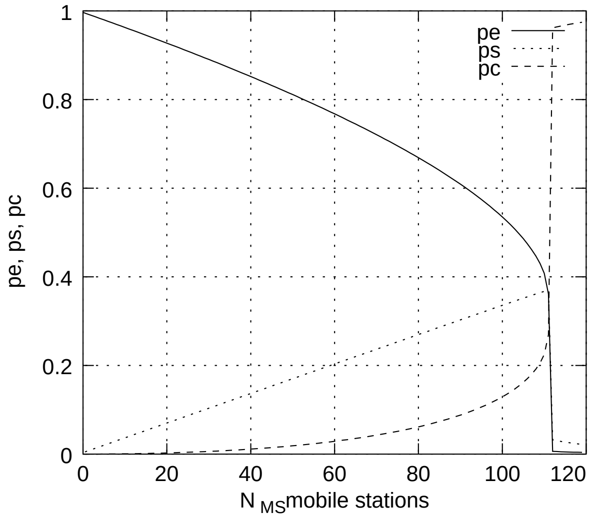
In this section we compare the results from the analytical analysis to performance measures based on stochastic simulation. We start with a description of the scenarios used for a performance evaluation of TETRAPOL systems. Then we give a short outline on the simulation concept. Finally, we present the comparing results.

5.1 ETSI scenarios

In [12] ten different scenarios are defined for the comparison of TETRA systems. For each scenario, detailed specifications have been laid down concerning speech activity and offered data traffic of the mobile end user. Furthermore, the channel model to be used, the size of the scenario area, the number and type of the mobile stations, mobile or hand radio



(a) $\lambda T_{RACH} = 0.002$ *and* $N_{MS} \in [1, 190]$



(b) $\lambda T_{RACH} = 0.00335$ *and* $N_{MS} \in [1, 120]$

Figure 9: Probabilities p_e, p_s, and p_c versus number of mobile stations N_{MS}

Table 1: Scenario 10 – General Parameters

Parameter	Value
Type of area	Bad Urban (BU)
Covered area	50 km^2
Subscriber density	50 km^{-2}
Subscriber distribution	Gaussian
Class of terminals	80% portable, 20% vehicle
Velocity	3–50 km/h
Grade of Service	5%

Table 2: Scenario 10 – Traffic per radio user

Parameter	Value
Speech activity	$A_s = 20\,\mathrm{mE}$
Call duration	$\bar{\beta}_s = 20\,\mathrm{s}$
Mean waiting time	$\bar{T}_w = 4\,\mathrm{s}$
Speech arrival rate	$\lambda_s = 3.6\,\mathrm{h}^{-1}$
Point-to-point connections	60%
Point-to-multipoint conn.	40%
Short data (100 byte)	$\lambda_{sd} = 20\,\mathrm{h}^{-1}$
Middle data (2 kbyte)	$\lambda_{md} = 0.5\,\mathrm{h}^{-1}$

terminal, and their maximum velocity have been defined. Due to the fact, that scenario 10 defines the highest amount of offered load per terminal, this scenario has been chosen for the performance analysis in this section. Scenario 10 describes the parameters of a public or private network for airlines ground services, airport security, fire brigades and so on. Table 1 depicts the general parameters defined by scenario 10.

5.2 TETRAPOL scenarios

Due to the absence of scenarios for the comparison of TETRAPOL systems or TETRA and TETRAPOL systems, our performance evaluation is based on ETSI scenario 10. As can bes seen from Table 2, the interarrival rate of speech connection set-up requests per mobile station is $\lambda_s = 3.6\,\mathrm{h}^{-1}$.

The speech arrival rate λ_s has been calculated using $\lambda_s = A_s/\bar{\beta}_s$. The mean waiting time is defined as the duration between the dialing of a subscriber or group number and the successful completion of the call set-up.

Furthermore, mobile stations are expected to send 100 byte short data messages at a rate $\lambda_{sd} = 20\,\mathrm{h}^{-1}$ and 2 kbyte middle data messages at a rate of $\lambda_{md} = 0.5\,\mathrm{h}^{-1}$. The total

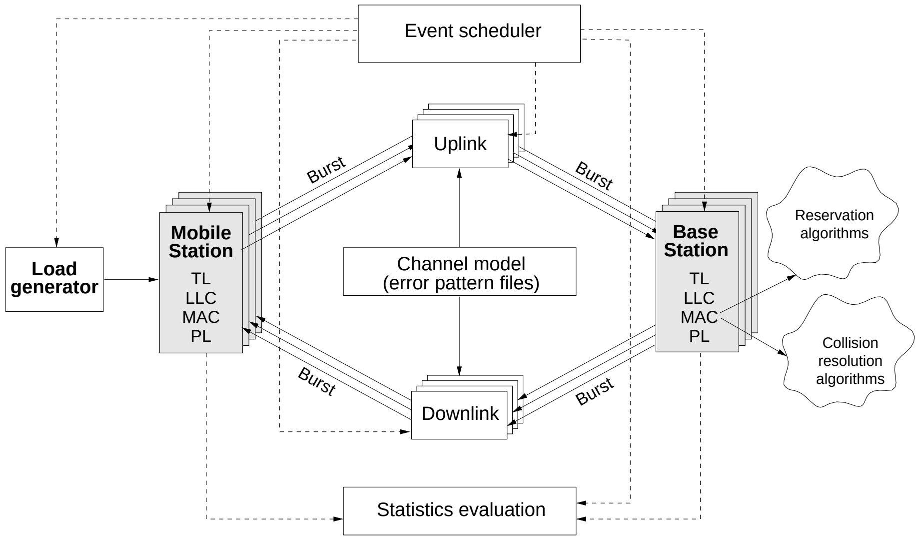


Figure 10: Structure of the simulation environment

sum λ of all arrival-rates per mobile station is:

$$\begin{aligned} \lambda &= \lambda_s + \lambda_{sd} + \lambda_{md} = 24.1\,\mathrm{h}^{-1} \\ \Rightarrow \lambda T_{RACH} &= 0.00335 \end{aligned} \tag{7}$$

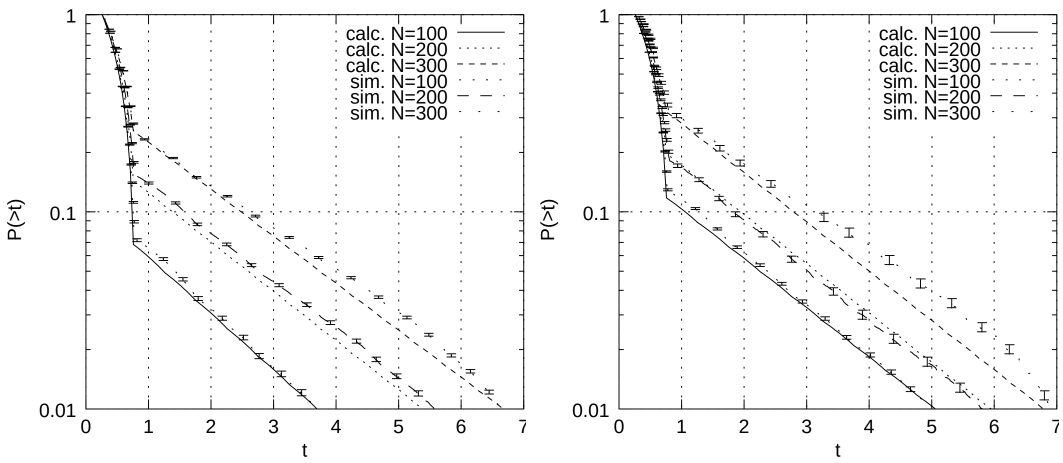
If $\lambda = 14.4\,\mathrm{h}^{-1}$ is assumed, λT_{RACH} equals 0.002. Each time, the mobile station initiates or re-establishes an application transaction, it uses the random access protocol. Hence, these arrival-rates have been used in the analytical discussions in the previous sections. The performance evaluations by simulation, which are described in this section, use the same traffic parameters.

5.3 Simulation concept

For the traffic performance evaluation of the TETRAPOL protocol stack the protocols of the air interface at reference point R_3 (see Section 2) have been implemented.

The structure of the simulator TETRIS is depicted in Figure 10. The protocol stack of the TETRAPOL system has been specified with the help of *Formal Description Techniques* (FDT) to guarantee not only syntactically and semantically unambiguous formal descriptions of the communication protocols but also interoperable and compatible implementations of these protocols independent of their implementors [14]. The *Specification and Description Language* (SDL) is the most widely used FDT in the area of telecommunications [17, 6, 9, 4, 20]. With the help of the C++ code generator SDL2SPEETCL [1], which converts SDL phrase representation to C++ source code, the mobile and base station protocol stacks have been embedded in a C++ simulation environment. The C++ implementations are based on the *SDL Performance Evaluation Tool Class Library* (SPEETCL) [1]. The SPEETCL provides generic C++ classes as well as a simulation library with strengths in random number generation, statistics evaluation, and event-driven simulation control.

The core of the simulator is the simulation control, which creates mobile and base stations and assigns the traffic generators to create specific traffic loads to the individual mobile stations. Depending on the scenario, the traffic generators are controlled to offer a certain traffic load. Traffic load is defined by inter-arrival times and the size of data units. The SPEETCL contains traffic load generators for applications like speech [3], video [8],



(a) $\lambda T_{RACH} = 0.002$ (b) $\lambda T_{RACH} = 0.00335$

Figure 11: Simulated and calculated complementary distribution function of the random access delay

Hypertext Transfer Protocol (HTTP) [2], *File Transfer Protocol* (FTP) [11], *Simple Mail Transfer Protocol* (SMTP) [10], and *Wireless Application Protocol* (WAP).

Mobile and base stations communicate via uplink and downlink channels, created by the simulation control, by exchanging bursts. With the help of error pattern files, transmission errors can be introduced on the uplink or downlink dependent on the actual *Carrier to Interference Ratio* (CIR) value at the respective receiver. The error pattern files have been generated taking into account a channel propagation model, the characteristics of the TETRAPOL physical layer, and the receiver characteristics.

5.4 Performance measurements

On basis of the analytical results presented in Section 4 and the simulation results, Figure 11 depicts the complementary distribution function of the random access delay. Additionally, 95% confidence intervals are shown, which have been calculated by means of the *Limited Relative Error* (LRE) algorithm [13]. The mean waiting time $\bar{T}_W = 4\,\mathrm{s}$ as claimed in the airport scenario is not exceeded. As both, TETRA and TETRAPOL, are waiting systems, the waiting time consists of the random access delay and the waiting time for a free traffic channel. The calculation of the waiting time for a free traffic channel is out of scope of this paper.

As can be seen from Tables 3, the analytical results for the mean waiting time and the mean throughput correspond to the results based on stochastic simulation with only small differences. As with $\lambda T_{RACH} = 0.002$ up to $3 \cdot N_{MS} = 558$ mobile station can be served by a base station utilizing one control channel, the maximum throughput is not reached for $3 \cdot N_{MS} = 400$. But as the mean waiting time rises more than the traffic offered, a network dimensioning strategy has be defined to find an optimum tradeoff between throughput and waiting time.

The analytical results give good approximations of the quality of service parameters in comparison to stochastic simulation. Hence, the analytical model can also be used to study different scenarios.

Table 3: Simulated and calculated mean random access delay and mean throughput

	$\lambda T_{RACH} = 0.002$				$\lambda T_{RACH} = 0.00335$			
	$E[T_W]$		$\bar{S}$		$E[T_W]$		$\bar{S}$	
$3 \cdot N_{MS}$	calc.	sim.	calc.	sim.	calc.	sim.	calc.	sim.
100	0.665	0.660	0.068	0.068	0.780	0.783	0.113	0.113
200	0.850	0.851	0.134	0.134	0.885	0.906	0.223	0.223
300	1.052	1.067	0.199	0.199	1.183	1.250	0.332	0.332
400	1.270	1.326	0.267	0.262	–	–	–	–

6 Comparison to TETRA

In [15, 16] a performance evaluation of the TETRA random access protocol is presented. Based on the airport scenario, ETSI scenario 10, the mean waiting time and mean throughput depending on the number of active *Mobile Stations* (MS) are calculated. In the case of a rolling access frame, a maximum number of $N_{MS} = 416$ mobile stations can be served, if the *Base Station* (BS) uses all four *Access Codes* (AC). If the BS uses only one AC, the mean throughput reaches its maximum for $N_{MS} = 556$ mobile stations. In case of a discrete access frame, $N_{MS} = 107$ MSs (four ACs) resp. $N_{MS} = 117$ MSs (one AC) can be served. Hence, the rolling access frame method is preferred over the discrete access frame method.

As discussed earlier, the maximum throughput of the random access protocol for the airport scenario in TETRAPOL systems is reached for $N_{MS} = 336$ mobile stations.

The mean access delay in TETRA systems is $E[T_W] = 709\,\text{ms}$ ($N_{MS} = 416$, four ACs) resp. $E[T_W] = 241\,\text{ms}$ ($N_{MS} = 556$, one AC). For TETRAPOL systems, we have measured a mean access delay of $E[T_W] \geq 1.25\,\text{s}$ for $N_{MS} = 336$ mobile stations (see Section 5.4). Thus, the performance of the TETRA random access protocol in terms of mean throughput and mean access delay is very much better than the one of TETRAPOL.

7 Conclusions

In this paper a Markovian model of the TETRAPOL random access protocol has been developed in order to determine the mean throughput and the complementary distribution function of the random access delay in typical traffic load situations. The maximum number of active users, which can be served by a base station, has been calculated and compared to a TETRA system.

Simulation results correspond to the values of the analytical results with only small differences. The results are used to derive quality of service parameters, e.g., mean throughput, waiting probability, and mean random access delay. In case of one control channel and a typical traffic load, up to approx. 300 mobile stations can be served by one TETRAPOL base station.

In comparison to the TETRA random access protocol, the performance of the multiple access scheme in TETRAPOL is very much weaker. The maximum number of mobile stations per base station is lower and the mean access delay is higher in TETRAPOL systems.

To serve a comparable number of active users in a TETRAPOL radio cell, the number of control channels has to be increased to compete against TETRA.

The analytical model described in this paper is suitable to determine quality of service parameters for different traffic load situations needed for TETRAPOL radio network dimensioning. Thus, the number of control channels needed for a given traffic load offered can be determined. It can also be used to optimize the parameters of the random access protocol for specific traffic load conditions regarding the number of wait state levels N_{max} and the Slotted-ALOHA access probability p_r.

References

[1] AixCom GmbH. http://www.aixcom.com. 5.3

[2] Tim Berners-Lee, R. Fielding, and H. Frystyk. Hypertext Transfer Protocol (HTTP) 1.0. Network Working Group, Request For Comments (RFC) 1945, May 1996. 5.3

[3] Paul T. Brady. A model for generating on–off speech patterns in two–way conversation. *The Bell System Technical Journal*, 48(9):2445–2472, September 1969. ISSN 0005–8580. 5.3

[4] Jan Ellsberger, Dieter Hogrefe, and Amardeo Sarma. *SDL – Formal Object–oriented Language for Communicating Systems*. Prentice Hall, 1997. ISBN 0–13–621384–7. 5.3

[5] B. Haverkort. *Performance of Computer Communication Systems: A Model-Based Approach*. John Wiley and Sons, Princeton, N.J., 1998. ISBN 0–471–97228–2. 4.1, 4.1

[6] ITU, Telecommunication Standardization Sector of. Specification and Description Language (SDL). ITU-T Recommendation Z.100, International Telecommunication Union, March 1993. 5.3

[7] J. Kemeny and L. Snell. *Finite Markov Chains*. Van Nostrand, Princeton, N.J., 1976. ISBN 3–540–90192–2. 4.1, 4.2

[8] Joan L. Mitchell, William B. Pennebaker, Chad E. Fogg, and Didier J. LeGall, editors. *MPEG Video Compression Standard*. Digital Multimedia Standards Series. Chapman and Hall, 1997. ISBN 0–201–54435–0. 5.3

[9] Anders Olsen, Ove Færgemand, Birger Møller-Pedersen, Rick Reed, and J.R.W. Smith. *Systems Engineering Using SDL–92*. Elsevier Science Publishers, 1994. ISBN 0–444–89872–7. 5.3

[10] Jonathan B. Postel. Simple Mail Transfer Protocol. Network Working Group, Request For Comments (RFC) 821, August 1982. 5.3

[11] Jonathan B. Postel and J. Reynolds. Telnet Protocol Specification. Network Working Group, Request For Comments (RFC) 854, May 1983. 5.3

[12] RES06. Scenarios for Comparison of Technical Proposals for MDTRS. Working Document (91) 23, ETSI, June 1991. 5.1

[13] Friedrich Schreiber. Effective Control of Simulation Runs by a New Evaluation Algorithm for Correlated Random Sequences. In *International Journal of Electronics and Communications (AEÜ)*, number 6 in 42, pages 347–354, November 1988. ISSN 0001-1096. 5.4

[14] Martin Steppler. Performance Analysis of Communication Systems Formally Specified in SDL. In *Proceedings of The First International Workshop on Simulation and Performance '98 (WOSP '98)*, pages 49–62, Santa Fe, New Mexico, USA, 12th–16th October 1998. 5.3

[15] Martin Steppler. Maximum Number of Users Which Can Be Served by TETRA Systems. In Bernhard Walke, editor, *Tagungsband der European Wireless '99*, volume 157 of *Reihe ITG-Fachbericht*, pages 315–320, München, 6th–8th Oktober 1999. VDE-Verlag. 6

[16] Martin Steppler and Michael Büter. Collision Resolution for TETRA Systems. In Bernhard Walke, editor, *Proceedings of 2. EPMCC '97*, volume 145 of *ITG-Fachbericht*, pages 559–565. Informationstechnische Gesellschaft (ITG) im Verein Deutscher Elektrotechniker (VDE), VDE-Verlag GmbH, 30th September–2nd Oktober 1997. ISBN 3-8007-2307-7. 6

[17] Martin Steppler and Matthias Lott. SPEET—SDL Performance Evaluation Tool. In Ana Cavalli and Amardeo Sarma, editors, *Proceedings of the 8th SDL Forum '97*, pages 53–67, Evry, France, 23rd–26th September 1997. Elsevier Science Publishers. ISBN 0-444-82816-8. 5.3

[18] TETRAPOL FORUM. TETRAPOL Specification: Air Interface Protocol: Air Interface Transport Protocol. Public Available Specification PAS 0001-3-3, TETRAPOL FORUM, Bois d'Arcy, France, January 1998. 3, 5, 3, 4.1

[19] TETRAPOL FORUM. TETRAPOL Technical Report: Guide to TETRAPOL features: References and Terminology. Public Available Specification TTR 0001-1-2, TETRAPOL FORUM, Bois d'Arcy, France, January 1998. 1, 2.1

[20] B. Walke. *Mobile Radio Networks, 2nd Edition*. John Wiley & Sons Ltd., West Sussex, England, 2001. ISBN 0-471-97595-8. 5.3

[21] Waterloo Maple Inc. `http://www.maplesoft.com`. 4.1