Error Probabilities for Radio Transmissions of MC-CDMA based W-LANs

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***Abstract*— IEEE 802.11a/e has become a worldwide wireless local area network (W-LAN) standard, with a rapid development. Many proposals have been made for its further expansion, and some of them focus on multicarrier code division multiple access (MC-CDMA), a novel, high capacity, multicarrier modulation scheme. In this paper we present an analysis of the error models for radio transmissions in such systems. An accurate model of the channel is necessary for the performance evaluation of the protocol by means of computer simulations. Focusing on the estimation of the signal to interference and noise ratio (SINR) at the detector and on the calculation of a packet error ratio (PER), in this contribution we discuss a modeling approach which allows an efficient calculation of frame transmissions over a MC-CDMA shared radio channel.**

Keywords-component; IEEE 802.11a/e; MC-CDMA; Radio Channnel Modeling; MMSE; PER; convolutional encoder;

I. INTRODUCTION

Multicarrier code division multiple access (MC-CDMA) has gained recently significant attention and has become a promising candidate for future wireless high capacity communication networks. Multicarrier techniques are generally robust against multipath fading, provide high spectrum efficiency and interference rejection capabilities. MC-CDMA has several other advantages, such as spectral diversity and immunity against frequency selective fading and impulse noise [4]. Due to the fast Fourier transform (FFT) associated with orthogonal frequency division multiplexing (OFDM), MC-CDMA chips are long in time duration, but narrow in bandwidth [8]. Consequently interchip interference is reduced, and synchronization is easier compared to other spread spectrum techniques.

Each symbol of the data stream of one user is multiplied by each element of the same spreading code and is thus placed in several narrow band subcarriers. Multiple chips are not sequentional, but transmitted in parallel on different subcarriers [8]. In MC-CDMA one single data symbol is spread in frequency [3]. Such a system with spreading factor (SF) four is presented in Fig. 1.

In the next section of this paper a calculation is given for the estimation of the signal to interference and noise ratio (SINR) at the detectors for MC-CDMA based asynchronous packet radio transmissions in Rayleigh fading environments. In an asynchronous multi-access MC-CDMA system like the one presented in [1], the received signal consists of all active users' information. This timing mismatch destroys the orthogonality of different users' spreading codes leading to multiple access interference (MAI) [4]. For this reason in [1] a linear multiuser detector (MUD) is applied at the receiver's side based on the minimum mean square-error (MMSE) criterion. A MMSE receiver combines both good performance and simplicity of implementation.

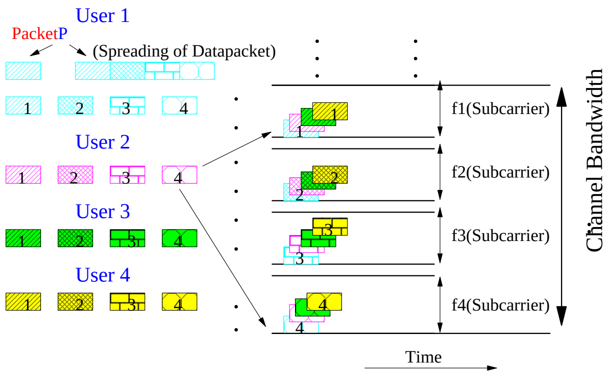

Figure 1. MC-CDMA principle with SF= 4

In Section III, this paper is concerned with the calculation of the packet error probability in multicarrier packet radio systems which utilize binary convolutional coding with hard decision Viterbi decoding. As defined in the standard [9], a binary convolutional encoder with constraint length K=7 and code rate R=½ is used to protect the transmitted packets from channel errors. Different code rates can be derived from the same convolutional encoder with puncturing, a procedure in which some of the coded bits at the output of the encoder are deleted before transmission.

In section IV an extensive presentation of simulation results is provided for different modulation schemes, coding rates and packet size. The calculations and results presented here are used for the estimation of radio channel effects in the simulation tool MACNET-2, and the accurate performance evaluation of MC-CDMA based W-LANs [1].

II. ESTIMATION OF SIGNAL TO INTERFERENCE AND NOISE RATIO

In this paper we consider an asynchronous multiuser MC-CDMA System. b_k is the k-th users binary symbol, which is transmitted in parallel over M (= SF) subcarriers in form of chips. Each chip is multiplied by a different element c_{km} of the spreading sequence. The spreading sequences used belong to the family of orthogonal Walsh-Hadamard codes and for SF=4 are given as rows of the matrix:

$$\mathbf{H}_4 = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix} \quad (1)$$

The MC-CDMA modulation scheme can be implemented using an IFFT [3], in the same manner as OFDM. For a subcarrier spacing of $1/T$, where T denotes the symbol duration at the subcarrier, the scheme is similar to performing OFDM modulation to a direct sequence code division multiple access (DS-CDMA) signal.

In our model we assume each subcarrier to experience frequency non-selective time-varying fading. During the transmission of one symbol though, fading is assumed constant. Under these conditions the received signal can be described from the following equation:

$$r(t) = \sum_{k=1}^{K} \sqrt{a_k} b_k \sum_{m=1}^{M} c_{km} h_{km} e^{j2\pi(t-\tau_k)m/T} p(t-\tau_k) + \eta(t) \quad (2)$$

where K is the maximum number of active users, a_k the transmission power of the k-th user's symbol b_k , M the number of subcarriers, $p(t)$ a rectangular pulse over [0,T], τ_k the delay of the k-th user and $\eta(t)$ denotes the additive white Gaussian noise. The Rayleigh fading process for the m-th subcarrier and k-th user is represented as:

$$h_{km} = \beta_{km} e^{j\phi km} \quad (3)$$

with β_{km} a Rayleigh distributed and φ_{km} a uniform over $[0,2\pi)$ distributed variable. For the rest of the calculation we focus on the signal of user 1, without loss of generality, and therefore the delays τ_k are relative to this first user and modeled as uniform random variables in the interval [0,T).

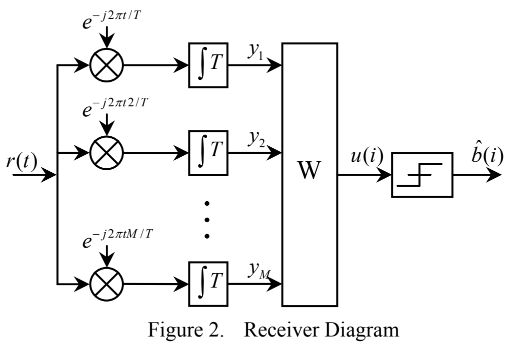

Figure 2. Receiver Diagram

As seen in the diagram of the MUD in Fig.2, after discarding the cyclic prefix and multicarrier demodulation by means of FFT [3], the component of the received signal at the m-th subcarrier can be expressed as [5]:

$$y_m = \frac{1}{T} \int_{T}^{(i+1)T} r(t) e^{-j2nmt/T} dt \quad (4)$$

In a linear multiuser detector the demodulator outputs y_m are multiplied with a decision variable w_m which is used for optimizing the decision of the detector on the transmitted symbol and mitigate the effects of the channel. Accordingly, the output of the linear MUD is [12]:

$$\hat{b}_k = \mathbf{w}^H \mathbf{y} \quad (5)$$

The optimum weight matrix $\mathbf{w}_{Mx1}$ for a given set of delays τ_K and fading parameters β_{km} is selected to minimize the mean square error of the detector:

$$MSE(\tau,\beta) = E\left\{ \left(\mathbf{w}^H y - b_k \right)^2 \right\} \quad (6)$$

Under these conditions the SINR can be calculated from the following equation proposed in [4], [5], and [12]:

$$SINR = \frac{\left| \sqrt{a_1} \mathbf{w}^H p_{K+1} \right|^2}{\mathbf{w}^H \Gamma \mathbf{w} + \left| \mathbf{w}^H \mathbf{P}_{K+1} \mathbf{A}_{\mathbf{K+1}} \right|^2} \quad (7)$$

where the matrices **P, p** are obtained from (4) as derived in the appendix. The first term in the denominator of (7) expresses the contribution of noise and the second the contribution of (MAI) to total interference.

In [4] the optimum weights are calculated in order to minimize the gradient of (7) with respect to **w,** leading to the maximum SINR (MSINR):

$$MSINR = a_1 \mathbf{p}_{K+1}^H (\mathbf{P}_{K+1} \mathbf{A}_{K+1}^2 \mathbf{P}_{K+1}^H \Gamma)^{-1} \mathbf{p}_{K+1} \quad (8)$$

At this point the importance of the cyclic prefix for the calculation of the PER should be considered. To avoid intersymbol interference (ISI) and to maintain orthogonality between the subcarriers in a time dispersive channel, a cyclic prefix is added to the multicarrier modulated symbol of duration T_S. ISI is avoided if the length of the cyclic prefix T_{CP} is selected large enough to exceed the maximum delay of the channel. We use a cyclic prefix of 800μs as proposed in [9] which has been proven to be sufficient especially in indoor wireless propagation channels. Since the receiver uses only the energy received during T_S , discarding the cyclic prefix, a power loss factor a_{CP} should be considered [13] depending on the relative length of the cyclic prefix divided by the total symbol duration:

$$a_{CP} = \frac{T_s}{T_s + T_{CP}} E_{av} \quad (9)$$

III. Calculation of PER

In this section we present a calculation of an upper bound for the PER on frame reception, based on the above estimation of the SINR and the convolutional decoder generator polynomials $g_1=133_8$ $g_2=171_8$. The calculation applies to both OFDM and MC-CDMA systems.

A. Symbol Error Probability

After the SINR is estimated, the symbol error probability for M-ary quadrature amplitude modulation (QAM) can be calculated [2]:

$$P_{SER,M-QAM} = 1-(1-P_{\sqrt{M}})^2 \tag{10}$$

$$P_{\sqrt{M}} = 2(1-\frac{1}{\sqrt{M}})Q(\sqrt{\frac{3}{M-1}\frac{E_{av}}{N_0}}) \tag{11}$$

The Q function is defined in [2] and E_{av} / N_0 is the average signal to noise ratio per symbol. The latter is estimated according to section II. For quadrature phase shift keying (QPSK) and binary phase-shift keying (BPSK) the symbol error rate (SER) is given by the following equation:

$$P_{SER,QPSK} = 2Q(\sqrt{\frac{E_{av}}{N_0}})\left[1-\frac{1}{2}Q(\sqrt{\frac{2E_{av}}{N_0}})\right] \tag{12}$$

$$P_{SER,BPSK} = Q(\sqrt{\frac{2E_{av}}{N_0}}) \tag{13}$$

B. Bit Error Probability

In the IEEE standard for OFDM physical layer [10], Gray coded constellation mapping is proposed. Thus the bit error rate (BER) for both QAM and QPSK (M=4) modulation can be calculated from the SER:

$$P_b = \frac{1}{\log_2 M} P_{SER,M} \tag{14}$$

For BPSK the BER is the same as the SER.

C. Packet Error Probability

The evaluation of the packet error probability is complicated by the fact that the errors in the decoder output stream are not independent. As described in [11] errors occur in bursts at the output of the Viterbi decoder, even if the errors into the decoder are independent. Thus an upper bound for the packet error probability is used which in this work is the tight bound derived in [11]:

$$P_e^m(L) \le 1-(1-P_u^m)^{8L}, \tag{15}$$

where L is the number of information bits in the packet and the union bound of first event error probability P_u^m is given by:

$$P_u^m = \sum_{d=d_{free}}^{\infty} a_d \cdot P_d \,, \tag{16}$$

with d_{free} being the minimum free distance of the convolutional code for the given code rate, a_d the total number of errors with weight d [12], [13] and P_d the probability of error in the pairwise comparison of two paths that differ in d bits. The values for a_d are obtained from the transfer function and represent the number of paths of distance d from the all-zero path. For the encoder used in this work the values are given in Table I.

TABLE I. Distance Spectra

D	a_d		
	code rate 1/2	***code rate 2/3***	***code rate 3/4***
d_{free}	11	1	8
$d_{free}+1$	0	16	31
$d_{free}+2$	38	48	160
$d_{free}+3$	0	158	892
$d_{free}+4$	193	642	4512
$d_{free}+5$	0	2435	23307
$d_{free}+6$	1331	6174	121077
$d_{free}+7$	0	34705	625059
$d_{free}+8$	7275	131585	3234886
$d_{free}+9$	0	499608	16753077
$d_{free}+10$	40406	values not relevant: $a_d P_d \to 0$	
$d_{free}+11$	0		
$d_{free}+12$	234969		
$> d_{free}+12$			

In case of hard decision decoding, the probability P_d can be calculated from the following equations:

$$P_d = \begin{cases} \sum_{k=(d+1)/2}^{d} \binom{d}{k} p^k (1-p)^{d-k}, & d = odd \\ \sum_{k=d/2+1}^{d} \binom{d}{k} p^k (1-p)^{d-k} + \binom{d}{\frac{d}{2}} \frac{p^{d/2}(1-p)^{d/2}}{2}, & d = even \end{cases} \tag{17}$$

where p is the bit error probability. Instead of the above expressions, the Chernoff upper bound can be used, leading to the following bound for the calculation of P_d:

$$P_d < [4p(1-p)]^{d/2} \tag{18}$$

IV. Numerical Results and Discussion

In this section we present the results of the previously presented analysis using different modulation and coding schemes. For that purpose we have chosen the most common used PHY-modes proposed for OFDM–based standards of IEEE [9] and ETSI/BRAN. An overview of the used PHY-modes and their basic characteristics is given in Table II. Further parameters of our MC-CDMA system are kept close to the values of the OFDM-based W-LAN standards [9] and can be taken from Table III.

TABLE II. PHY MODES

PHY-mode m	Modulation	Code rate	Data bits per symbol	d_{free}
1	BPSK	1/2	24	10
2	BPSK	3/4	36	5
3	QPSK	1/2	48	10
4	QPSK	3/4	72	5
5	16 QAM	1/2	96	10
6	16 QAM	3/4	144	5
7	64 QAM	2/3	192	6
8	64 QAM	3/4	216	5

TABLE III. SYSTEM PARAMETERS

Parameter	Value
Spreading Factor	4
Number of Subcarriers	48 Data + 4 Pilot
Subcarrier Spacing	0.3125 MHz
Channel Bandwidth	20 MHz
Carrier Frequency	5.25 GHz
Noise Level	-93dBm
Symbol Interval	4 µs = 3.2 µ + 0.8 µs
Guard Interval	0.8 µs
Preamble Length	16 µs

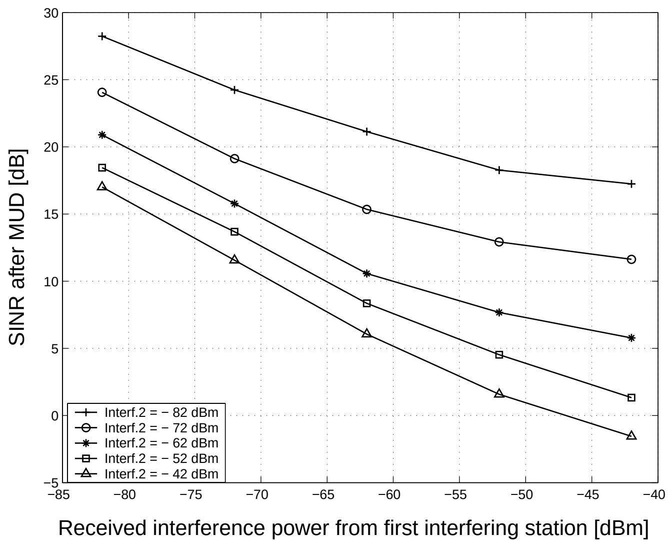

Figure 3. MUD performance with 4 active users and SF=4

In Fig. 3 the performance of the MUD is presented based on the previous analysis. Simulations were performed with 4 active users, modeling one carrier and 3 interfering signals. Two parameters are constant; the received carrier strength is set to -60 dBm and the power of the third interferer -55 dBm. Noise power is -93 dBm. The interference power received from the other interferers varies between -82 dBm (sensing range) and -42 dBm. Fig. 3 presents the SINR at the multiuser detector vs. the power of the first interferer for five different values of the second interferer's power. The SINR value on the diagram is the average over 10,000 runs with different delays τ_k. The graphs show that performance decreases almost linearly with the interfering power. One important observation is that the MUD manages to provide a positive SINR even in the case when all the three interfering signals are 5dB higher than the carrier strength.

Fig. 4 presents the PER vs. E_{av}/N_0 for 1514 byte long frames which is the maximum size of an Ethernet packet. The solid lines refer to multicarrier systems with a power loss due to the cyclic prefix of 80%. It is interesting to observe the performance of the BPSK ¾ PHY-mode in comparison to QPSK ½ which is very close. Thus it is better to use the QPSK ½ mode since more data bits are transmitted per symbol (Table II). In Fig. 5 the PER vs. E_{av}/N_0 is given for frame lengths of 48, 512, 2304 bytes and different PHY-modes. The maximum frame length used is 2304 byte, which the maximum payload of IEEE 802.11a. The diagrams show that PER is growing with the packet size.

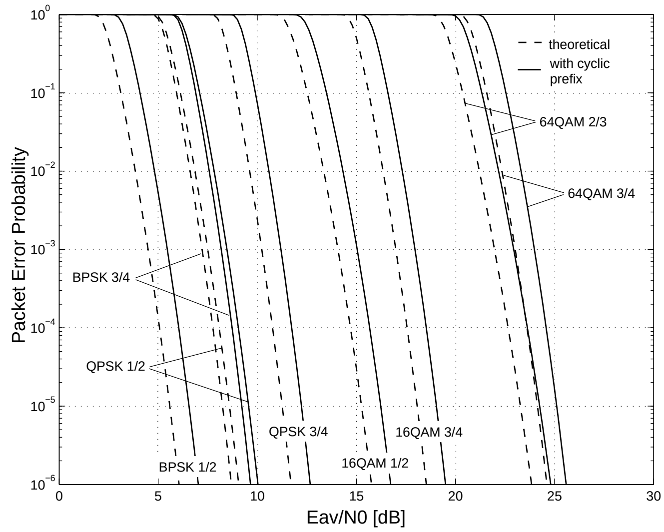

Figure 4. Theoretical PER for a 1514 byte long packet. The solid lines refer to a multicarrier system with a_{CP} = 80%.

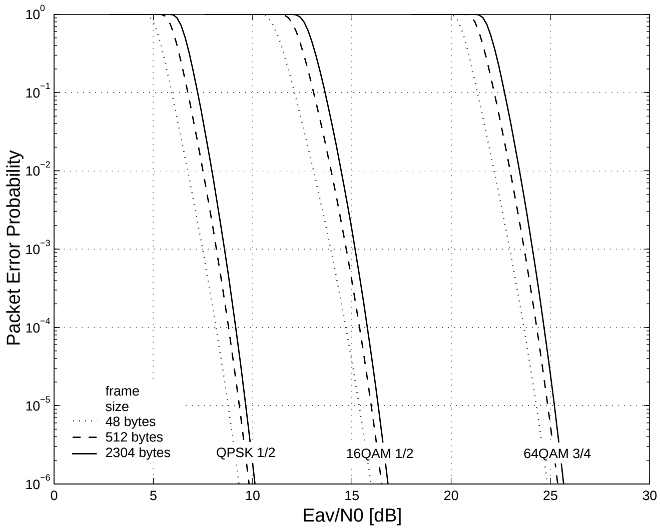

Figure 5. PER vs. E_{av}/N_0 for different packet sizes

V. CONCLUSIONS

In this paper we have presented a model which estimates the PHY-layer performance of a MC-CDMA based W-LAN. After computing the SINR at the detector, the model provides a calculation of the bound on the packet error probability for a packet radio system that employs convolutional encoding and hard decision Viterbi decoding. Calculations are done by taking into account the channel characteristics, parameters of the multicarrier spread spectrum scheme as well as different modulation and coding techniques. This model intends to support the calculation of packet transmission effects and has therefore been implemented in the MACNET-2 simulator, an SDL/C++ based simulation tool which models a MC-CDMA W-LAN based on the IEEE 802.11 MAC protocol [1].

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VI. APPENDIX

Eq. (4) can be expressed as:

$$y_n = \sum_{k=1}^{K} \sqrt{a_k}\left[p_{nk} b_k(i) + \overline{p}_{nk} b_k(i-1)\right] + \zeta_n \quad (19)$$

where:

$$p_{nk} = \frac{1}{T}\sum_{m=1}^{M} c_{km} h_{km} e^{-j2\pi n\tau_k/T} \int_{\tau_k}^{T} e^{j2\pi(m-n)t/T} dt \quad (20)$$

$$\overline{p}_{nk} = \frac{1}{T}\sum_{m=1}^{M} c_{km} h_{km} e^{-j2\pi n\tau_k/T} \int_{0}^{\tau_k} e^{j2\pi(m-n)t/T} dt \quad (21)$$

$$\zeta_n = \frac{1}{T}\int_0^T \eta(t) e^{-2\pi nt/T} dt \quad (22)$$

By analyzing eq. (20) a simpler form can be derived:

$$p_{nk} = \frac{1}{T}\sum_{m=1}^{M} \frac{c_{km}\beta_{km}(T-\tau_k)}{T}\left\{\cos\left[\left(\frac{2\pi n\tau_k}{T}\right) - \phi_{km}\right] - j\sin\left[\left(\frac{2\pi n\tau_k}{T}\right) - \phi_{km}\right]\right\} \quad (23)$$

which can be also applied to (21). Eq. (19) can now be expressed with matrix notation:

$$\mathbf{y(i)} = \mathbf{PAb(i)} + \zeta\mathbf{(i)} \quad (24)$$

with:

$$\mathbf{y(i)} = [y_1, \ldots y_k]^T$$
$$\mathbf{A} = diag[\sqrt{a_1}, \ldots, \sqrt{a_k}, \sqrt{a_1}, \ldots, \sqrt{a_k}] \quad (25)$$
$$\mathbf{b(i)} = [b_1(i-1), \ldots, b_k(i-1), b_1(i), \ldots, b_k(i)]$$

$$\mathbf{P} = \begin{bmatrix} \overline{p}_{11} & \cdots & \overline{p}_{1K} & p_{11} & \cdots & p_{1K} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \overline{p}_{M1} & \cdots & \overline{p}_{MK} & p_{M1} & \cdots & p_{MK} \end{bmatrix} \quad (26)$$

$$\zeta\mathbf{(i)} = [\zeta_1, \ldots, \zeta_M]^T \quad (27)$$

where ζ denotes a white Gaussian noise Vector with covariance matrix $\mathbf{\Gamma}_{MxM} = \sigma_n^2\,\mathbf{I}$.

For (7),(8), $\mathbf{P}_{K+1}$ is formed from $\mathbf{P}$ when omitting the K+1 column, $\mathbf{p}_{K+1}$ is the K+1 column of $\mathbf{P}$ and $\mathbf{A}_{K+1}$ is formed from $\mathbf{A}$ when omitting the K+1 column.