Fast Collision Resolution in Wireless ATM Networks*

Dietmar Petras, Andreas Krämling

Communication Networks, Aachen Univ. of Technology
E-Mail: {petras|akr}@comnets.rwth-aachen.de
WWW: http://www.comnets.rwth-aachen.de/~petras

Abstract — **The paper models the medium access control (MAC) layer of a wireless ATM network as a distributed queueing system. A random access channel with short slots is used for the transmission of capacity requests from distributed queues in the wireless terminals to the central scheduler in the base station. The paper describes the mathematical analysis of a fast collision resolution algorithm which is based on conventional splitting algorithms but employs identifiers of terminals to choose a subset after a collision. Based on the analytical results, a new medium access control protocol for the random access channel is defined which is called *probing algorithm*. Its performance is evaluated by stochastic simulations.**

1 Introduction

Future broadband multimedia telecommunication networks according to the I-300-series of the ITU-T recommendations are based on a packet switching technique established in 1990/91, the so-called asynchronous transfer mode (ATM).

In this paper we analyse a model of the medium access control (MAC) layer of a wireless (W) ATM network. The MAC layer is characterized by the realization of a distributed queueing system in Fig. 1 as described in [4, 5]. The scheduler of the distributed queueing system is located in the central base station. The buffers with packets (so-called ATM cells) waiting for transmission over the radio link from the wireless terminals to the base station are located in the terminals.

A difficult task of the MAC protocol is the transmission of the queue status (so-called capacity request) from the terminals to the scheduler in the base station, where it is required for the correct execution of the serving strategy of the scheduler.

Usual MAC protocols for W-ATM networks are using frames of variable length (so-called periods) with slots for the transmission of ATM cells and shorter slots for the transmission of capacity requests. At the beginning of each period the assignment of slots of the period to terminals is broadcasted by the base station. The number of short slots in a period can be chosen from 0 to n with realistic $n < 50$. The sequence of short slots is called random access channel (RACH). Polling means, that the base station invites a specific terminal to transmit in a reserved slot. Random access happens, if a group of or all terminals are allowed to transmit in a slot. The result of a random access (ternary feedback: free, successful, collision) is broadcasted by the base station at the end of each period over a feedback channel. An error-free feedback is assumed. In the conclusions of the paper we discuss the effect of faulty feedbacks. If a collision occurs, all collided packets are lost and have to be retransmitted[1]. A collision resolution algorithm is necessary to guaranty stability and limited delays [1].

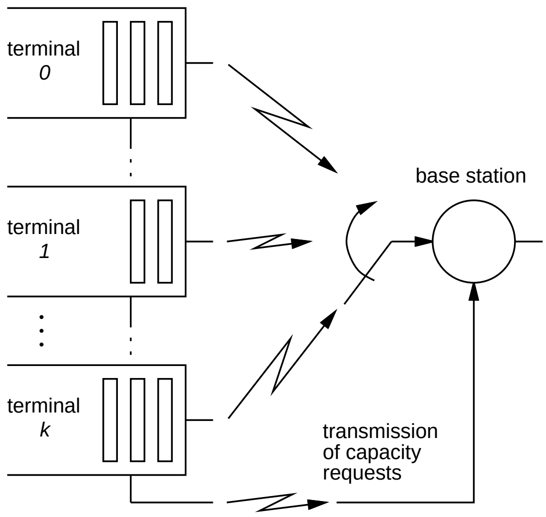

Figure 1: Modelling the MAC layer of a W-ATM network as a distributed queueing system

The literature describes splitting algorithms as the collision resolution algorithms with highest throughput [2, 3]. In this family of algorithms terminals are grouped to sets. All terminals of a set are allowed to transmit in a specific slot. A transmission will only be successful, if a set contains exact one terminal. After a collision the set is split into several subsets according to the order of the collision resolution algorithm (two subsets with binary algorithms, three subsets with ternary algorithms, etc). A collided terminal chooses its followup subset by using a certain strat-

*Supported by: The German Federal Ministry of Education, Science, Research and Technology

[1] Capture may enable the reception of the packet with the highest signal strength even if a collision occurred. This effect is neglected.

egy (eg. by an unbiased random experiment, so-called coin flipping). If no collision occurs in a subset, the collision is resolved, otherwise the subset is split again. In blocking algorithms, a collision resolution phase is started with a start set. Terminals with new arrivals are not allowed to access the subsets of an ongoing phase. They have to wait for the next start set founding a new phase. Unblocked algorithms allow new arrivals to enter the current phase directly. It has been shown that this decreases the performance slightly but reduces implementation effort [3]. The performance of collision resolution algorithms is given by throughput (radio of slots with successful transmission to all slots) and delays.

Due to the support of realtime oriented multimedia services in ATM networks, the collision resolution algorithm in a W-ATM network is less to be optimized to throughput but to short delays. Furthermore, the maximum delay is an important performance parameter.

2 Model of the RACH

We introduce the following model of the RACH: We assume that a period of duration τ_P is able to offer any number of RACH slots. In a terminal the need for transmitting capacity requests is modelled by the arrival of a packet. We assume the probability p of at least one new arrival in a terminal during one period τ_P. New arrivals can only occur at terminals with no waiting packet. At the start of each period the base station determines the assignment of slots to groups of terminals. At the end of each period the error-free feedbacks are broadcasted.

This model differs from that of other systems in following items:

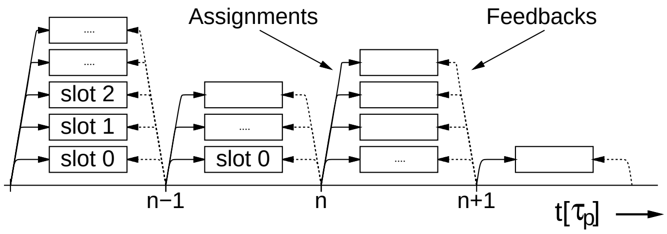

Figure 2: Model of the RACH for transmission of capacity requests

- Limited and known number of terminals, since terminals have to register before transmitting capacity requests. The terminals are numbered by consecutive identifiers from a identifier space $[0, \ldots, o^n - 1]$ with o being the order of the space and n being its dimension.
- Unlimited number of simultaneous slots per period
- Delayed feedback at the end of each period
- Delays are measures as multiples of the period duration τ_P.

3 Analysis of the Identifier Splitting Algorithm

Since the number of terminals is limited and known, it is useful to distribute collided terminals on the followup subsets according to their identifiers leading to the *identifier splitting algorithm*. With each splitting step the dimension n of the remaining identifier space decreases by one.

For the binary ($o = 2$) identifier splitting algorithm the number of terminals in the resulting subsets is a hypergeometric random variable. In case of a collision ($k \geq 2$) the probability of k_l terminals choosing the left subset and the remaining $k_r = k - k_l$ terminals choosing the right subset is:

$$P_{n,k}(k_l) = \frac{\binom{2^{n-1}}{k_l}\binom{2^{n-1}}{k-k_l}}{\binom{2^n}{k}} \tag{1}$$

We analyse the throughput of the identifier splitting algorithm by determining the number of slots $N_{o,n}(k)$ for the resolution of a start set of k terminals (splitting order o, dimension n of identifier space). The recursions (2) for the binary and (3) for the ternary algorithm are used with the starting condition $N_{o,n}(0) = N_{o,n}(1) = 1$.

$$\begin{aligned} N_{2,n}(k) &= 1 + \sum_{i=\max(0,k-2^{n-1})}^{\min(k,2^{n-1})} \frac{\binom{2^{n-1}}{i}\binom{2^{n-1}}{k-i}}{\binom{2^n}{k}} \Big(N_{2,n-1}(i) + N_{2,n-1}(k-i) \Big) \\ &= 1 + \frac{2}{\binom{2^n}{k}} \sum_{i=\max(0,k-2^{n-1})}^{\min(k,2^{n-1})} \binom{2^{n-1}}{i}\binom{2^{n-1}}{k-i} N_{2,n-1}(i) \end{aligned} \tag{2}$$

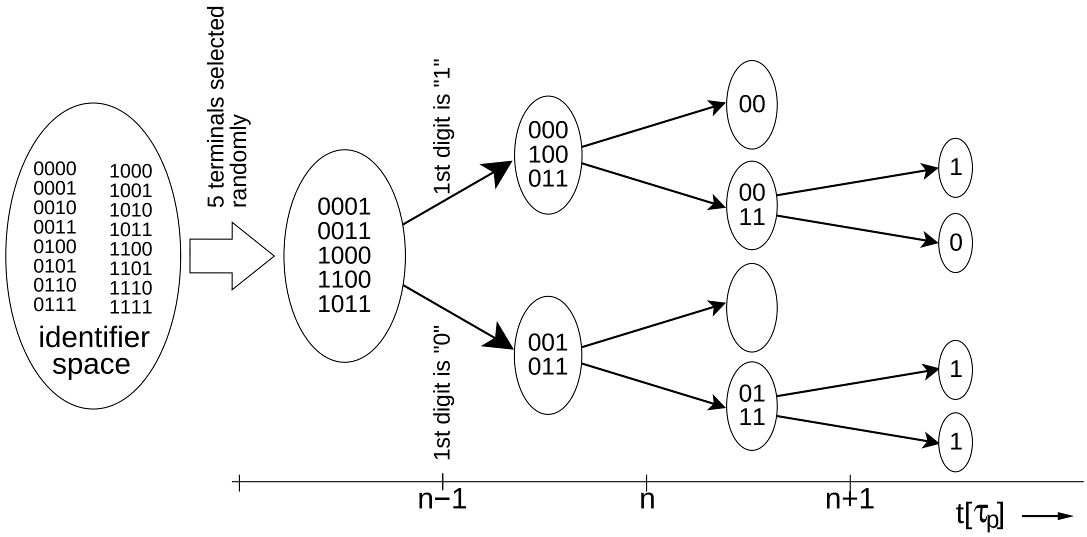

Figure 3: Example of binary ($o = 2$) identifier splitting with identifier space of dimension $n = 4$

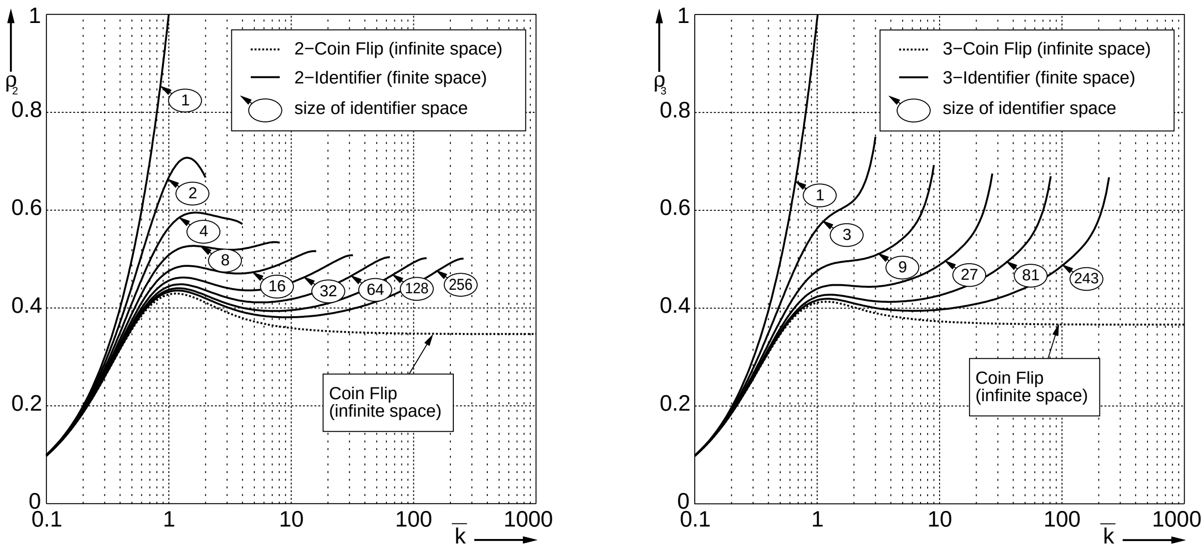

Figure 4: Throughput $\rho_{o,n}(\overline{k})$ of binary ($o = 2$) and ternary ($o = 3$) identifier splitting algorithm with binomial-distributed number of terminals in a start set

$$\begin{aligned} N_{3,n}(k) &= 1 + \sum_{i=0}^{\min(k,3^{n-1})} \sum_{j=\max(0,k-i-3^{n-1})}^{\min(k-i,3^{n-1})} \frac{\binom{3^{n-1}}{i}\binom{3^{n-1}}{j}\binom{3^{n-1}}{k-i-j}}{\binom{3^n}{k}} \Big(\cdots \\ &\qquad \cdots N_{3,n-1}(i) + N_{3,n-1}(j) + N_{3,n-1}(k-i-j) \Big) \end{aligned} \tag{3}$$

The size k of a start set of a collision resolution phase is a binomial random variable with $0 \leq k \leq o^n$ and mean $\overline{k}$. Thus, the throughput $\rho_{o,n}(\overline{k})$ is calculated by (4).

$$\rho_{o,n}(\overline{k}) = \frac{\overline{k}}{\sum_{k=0}^{o^n} N_{o,n}(k) \cdot \binom{o^n}{k} \left(\frac{\overline{k}}{o^n}\right)^k \left(1 - \frac{\overline{k}}{o^n}\right)^{o^n - k}} \tag{4}$$

The curves $\rho_{o,n}(\overline{k})$ in Fig. 4 for the binary and ternary algorithm have been calculated numerically. For comparison, the throughput of the coin flip splitting algorithm with a Poisson distributed size of a start set is given.

The distribution of delays is also calculated by a recursion. We define the probability $p_{o,n,k}(l, m)$ of m mobiles still being involved in a collision of a start set of k terminals after l splitting steps.

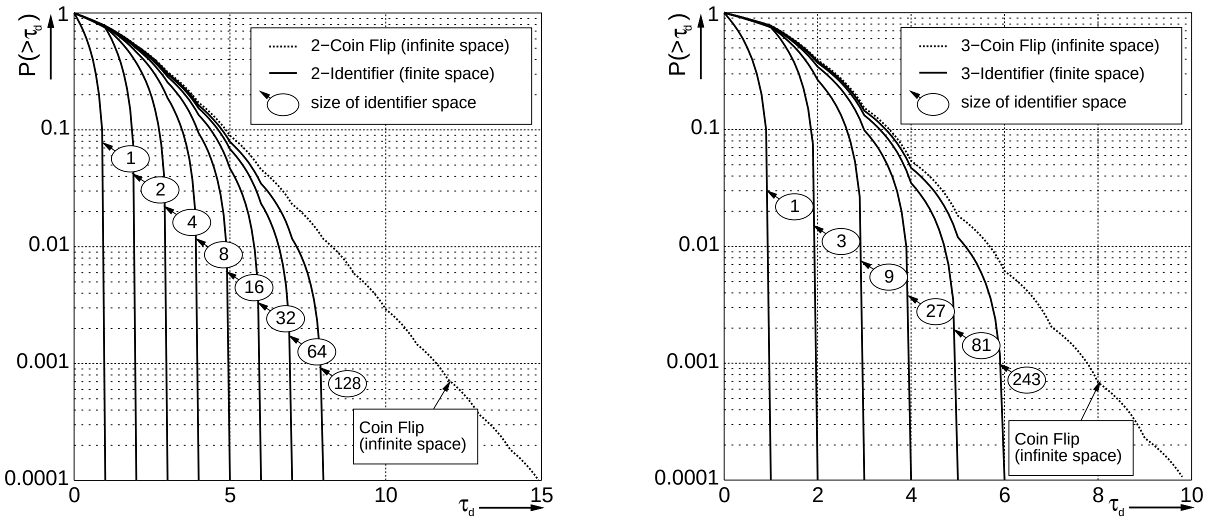

Figure 5: Complementary distribution of delays $P_{o,n,\overline{k}}(\tau_d > t)$ of binary ($o = 2$) and ternary ($o = 3$) identifier splitting algorithm with binomial distributed number of terminals in a start set (operating point $\overline{k} = 1.5$)

$$p_{2,n,k}(l,m) = \begin{cases} 1 & \textit{for} \quad m = k, l = 0 \\ 1 & \textit{for} \quad m = 0, k \leq 1, l > 0 \\ \displaystyle\sum_{i=\max(0,k-2^{n-1})}^{\min(k,2^{n-1})} \frac{\binom{2^{n-1}}{i}\binom{2^{n-1}}{k-i}}{\binom{2^n}{k}} \sum_{j=\max(0,m-(k-i))}^{\min(m,i)} p_{2,n-1,i}(l-1,j) \cdot p_{2,n-1,k-i}(l-1,m-j) & \\ & \textit{for} \quad k > 1, l > 0 \\ 0 & \textit{else} \end{cases} \tag{5}$$

$$p_{3,n,k}(l,m) = \begin{cases} 1 & \textit{for} \quad m = k, l = 0 \\ 1 & \textit{for} \quad m = 0, k \leq 1, l > 0 \\ \displaystyle\sum_{i=0}^{\min(k,3^{n-1})} \sum_{j=\max(0,k-i-3^{n-1})}^{\min(k-i,3^{n-1})} \frac{\binom{3^{n-1}}{i}\binom{3^{n-1}}{j}\binom{3^{n-1}}{k-i-j}}{\binom{3^n}{k}} \sum_{r=0}^{\min(m,i)} \sum_{s=\max(0,m-r-(k-i-j))}^{\min(m-r,j)} \cdots & \\ \quad \cdots p_{3,n-1,i}(l-1,r) \cdot p_{3,n-1,j}(l-1,s) \cdot p_{3,n-1,k-i-j}(l-1,m-r-s) & \\ & \textit{for} \quad k > 1, l > 0 \\ 0 & \textit{else} \end{cases} \tag{6}$$

The complementary distribution of the delays τ_d results in:

$$P_{o,n,k}(\tau_d > t) = \frac{1}{k} \sum_{m=0}^{k} m \cdot p_{o,n,k}(\lfloor t/\tau_P \rfloor, m) \quad , \quad k > 0 \tag{7}$$

Taking into account the binomial distributed size $\overline{k}$ of a start set, we get the complementary distribution of delays $P_{o,n,\overline{k}}(\tau_d > t)$ in (8). Fig. 5 shows the curves with the numerically calculated values.

$$P_{o,n,\overline{k}}(\tau_d > t) = \frac{1}{\overline{k}} \sum_{k=0}^{o^n} \binom{o^n}{k} \left(\frac{\overline{k}}{o^n}\right)^k \left(1 - \frac{\overline{k}}{o^n}\right)^{o^n - k} \cdot k \cdot P_{o,n,k}(\tau_d > t) \quad , \quad k > 0 \tag{8}$$

The calculation of the average delay $\overline{\tau}_d$ is based on the probability function of the number of periods required for the successful transmission of a packet in eq. (9) and (10).

$$p_{2,n,k}(l) = \begin{cases} 1 & \textit{for} \quad k = 1, l = 1 \\ \displaystyle\frac{1}{k} \sum_{i=\max(0,k-2^{n-1})}^{\min(k,2^{n-1})} \frac{\binom{2^{n-1}}{i}\binom{2^{n-1}}{k-i}}{\binom{2^n}{k}} \left(i \cdot p_{2,n-1,i}(l-1) + (k-i) \cdot p_{2,n-1,k-i}(l-1)\right) & \\ & \textit{for} \quad k > 1, l > 1 \\ 0 & \textit{else} \end{cases} \tag{9}$$

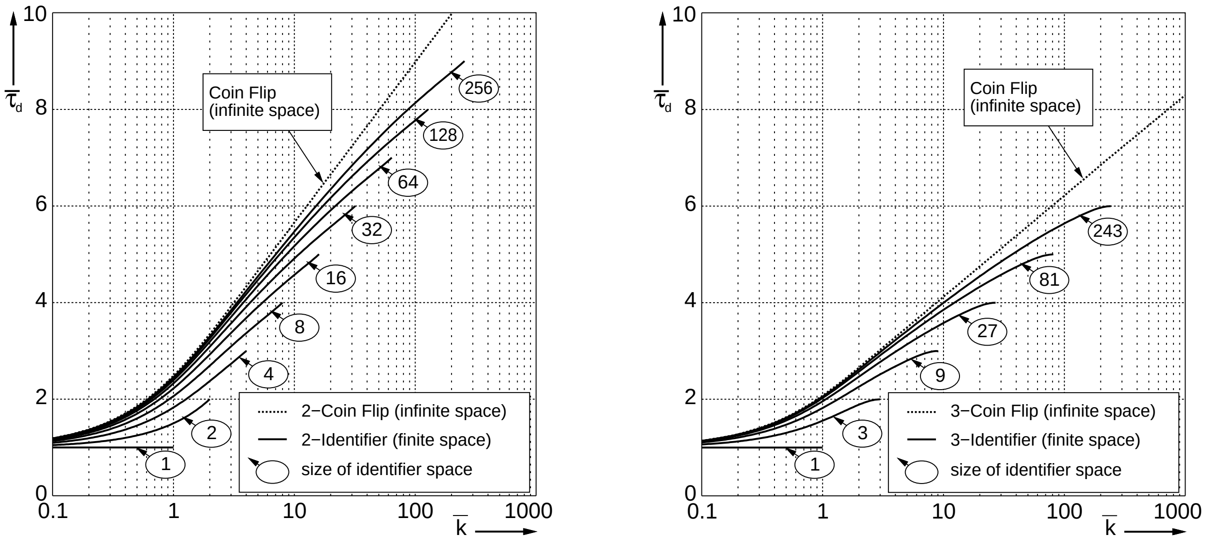

Figure 6: Average delay $\overline{\tau}_{d\,o,n}(\overline{k})$ of binary ($o = 2$) and ternary ($o = 3$) identifier splitting algorithm with binomial distributed number of terminals in a start set

$$p_{3,n,k}(l) = \begin{cases} 1 & \textit{for} \quad k = 1, l = 1 \\ \frac{1}{k} \sum\limits_{i=0}^{\min(k,3^{n-1})} \sum\limits_{j=\max(0,k-i-3^{n-1})}^{\min(k-i,3^{n-1})} \frac{\binom{3^{n-1}}{i}\binom{3^{n-1}}{j}\binom{3^{n-1}}{k-i-j}}{\binom{3^n}{k}} \Big(\cdots & \\ \quad \cdots i \cdot p_{3,n-1,i}(l-1) + j \cdot p_{3,n-1,j}(l-1) + (k-i-j) \cdot p_{3,n-1,k-i-j}(l-1) \Big) & \\ & \textit{for} \quad k > 1, l > 1 \\ 0 & \textit{else} \end{cases} \tag{10}$$

Using these equations, the average delay $\overline{\tau}_{d\,o,n}(\overline{k})$ and its variance σ^2 can be calculated. The curves $\overline{\tau}_{d\,o,n}(\overline{k})$ are shown in Fig. 6.

$$\overline{\tau}_{d\,o,n}(\overline{k}) = \tau_P \cdot \frac{1}{\overline{k}} \sum_{k=0}^{o^n} \left(\binom{o^n}{k} \left(\frac{\overline{k}}{o^n}\right)^k \left(1 - \frac{\overline{k}}{o^n}\right)^{o^n-k} \cdot k \sum_{l=0}^{n+1} l \cdot p_{o,n,k}(l) \right) \quad , \quad k > 0 \tag{11}$$

$$\sigma^2\left(\overline{\tau}_{d,o}(\overline{k})\right) = \frac{1}{\overline{k}} \sum_{k=0}^{o^n} \left(\binom{o^n}{k} \left(\frac{\overline{k}}{o^n}\right)^k \left(1 - \frac{\overline{k}}{o^n}\right)^{o^n-k} \cdot k \sum_{l=0}^{n+1} (l \cdot \tau_P - \overline{\tau}_{d\,o,n}(\overline{k}))^2 \cdot p_{o,n,k}(l) \right) \quad , \quad k > 0 \tag{12}$$

4 Derivation of the Adaptive Identifier Splitting Algorithm

If the size of a start set is large, the throughput can be increased and delays reduced, if the first splitting steps are skipped. This is equivalent to a dynamically selected splitting order o^{n_1} of the first splitting step. That n_1 is chosen that maximizes throughput:

$$\rho_{opt\,o,n}(\overline{k}) = \max\left(\rho_{o,0}\left(\frac{\overline{k}}{o^n}\right), \cdots, \rho_{o,n-n_1}\left(\frac{\overline{k}}{o^{n_1}}\right), \cdots, \rho_{o,n}\left(\overline{k}\right) \right) \tag{13}$$

The resulting curves of $\rho_{opt\,o,n}(\overline{k})$ are shown in Fig. 7. The curves result from a piecewise composition of segments of the curves in Fig. 4. The same applies for the average delay $\overline{\tau}_{d_{opt\,o,n}}(\overline{k})$ in Fig. 8.

The exact ordinate values $\overline{k}$ of the transitions between segments can be calculated by (14).

$$\rho_{o,n-n_1-1}\left(\frac{\overline{k}}{o^{n_1+1}}\right) = \rho_{o,n-n_1}\left(\frac{\overline{k}}{o^{n_1}}\right) \tag{14}$$

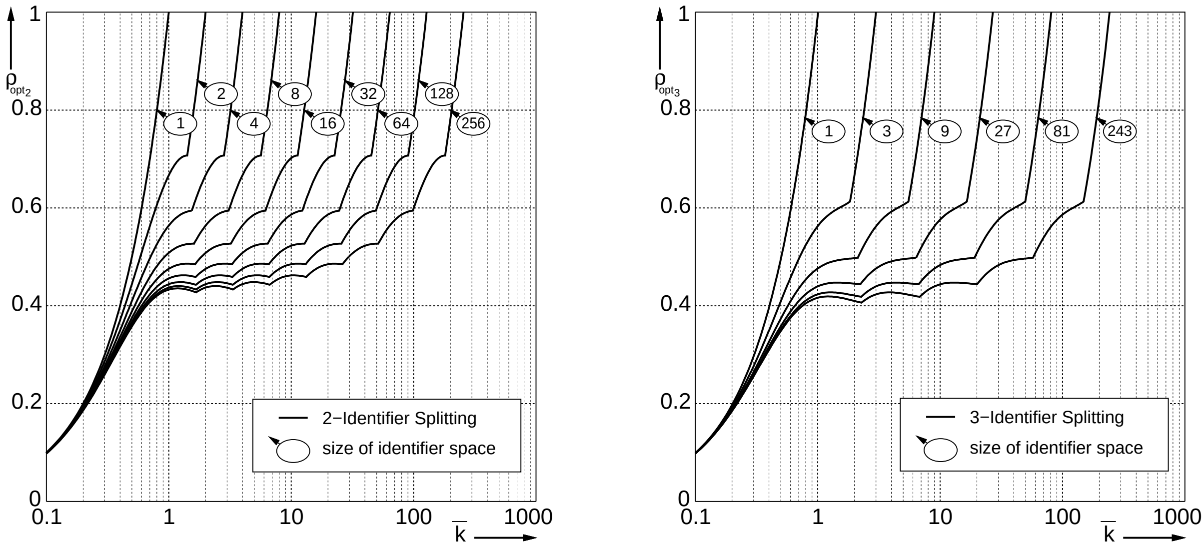

Figure 7: Optimal throughput $\rho_{opt_{o,n}}(\overline{k})$ of identifier splitting algorithm with adaptive order of first splitting step

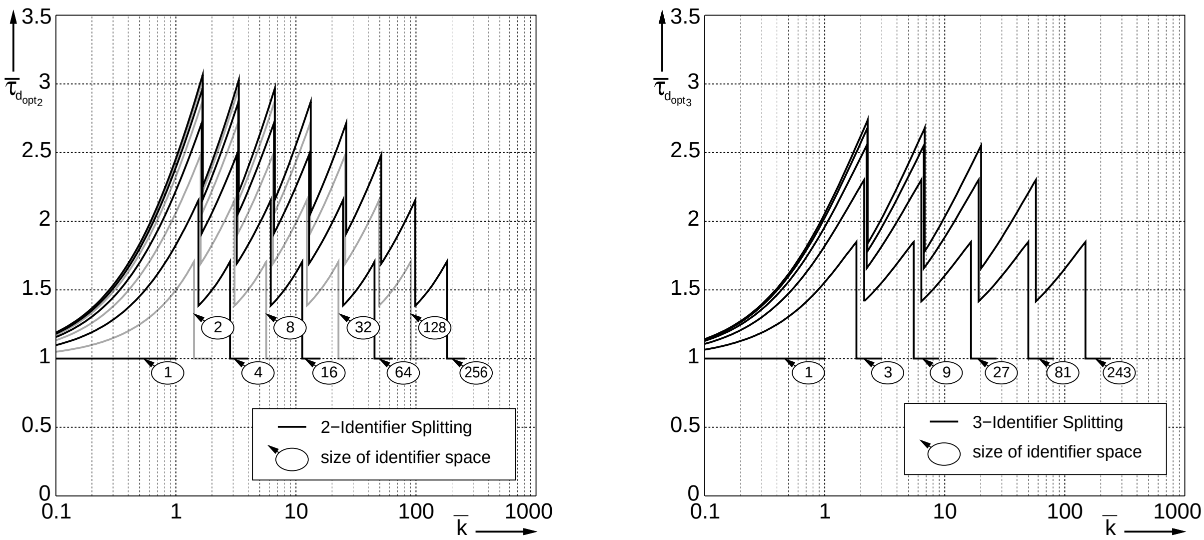

Figure 8: Optimal average delay $\overline{\tau}_{d_{opt_{o,n}}}(\overline{k})$ of identifier splitting algorithm with adaptive order of first splitting step

Table 1 summarizes the ordinate values for the binary and ternary algorithm. Dependent on the dimension of the identifier space and the known or estimated size $\overline{k}$ of a start set, the optimal number of initial slots can be determined.

The comparison of Fig. 7 and Fig. 8 of the adaptive identifier splitting algorithm with the corresponding Figures 4 and 6 of the original identifier splitting algorithm demonstrated the dramatical improvement of performance that can be realized, if the size of a start set is known or can at least be estimated.

5 Simulation of Medium Access Control Protocol with Probing Algorithm

The analysis of the identifier splitting algorithm with an adaptive number of initial slots has shown, that the optimal size of a start set is approximately 1.5 for binary splitting and 2 for ternary splitting with some deviations depending on the dimension of the identifier space. Now we return to the model of the RACH. We can estimate the probability $p_{\geq 1}$ of at least one arrival at terminal i during the interval $n_{idle,i} \cdot \tau_P$ since its last transmission of a packet:

Size of identifier space: 1	2	4	8	16	32	64	128	256	Number of initial slots
>0	>0	>0	>0	>0	>0	>0	>0	>0	1
	>1.4142	>1.5429	>1.6091	>1.6432	>1.6607	>1.6695	>1.6739	>1.6761	2
		>2.8284	>3.0858	>3.2181	>3.2865	>3.3214	>3.3390	>3.3478	4
			>5.6569	>6.1716	>6.4362	>6.5730	>6.6427	>6.6780	8
				>11.314	>12.343	>12.872	>13.146	>13.285	16
					>22.628	>24.687	>25.745	>26.292	32
						>45.255	>49.373	>51.490	64
							>90.510	>98.746	128
								>181.02	256

Size of identifier space: 1	3	9	27	81	243	Number of initial slots
>0	>0	>0	>0	>0	>0	1
	>1.8391	>2.1306	>2.2352	>2.2711	>2.2832	3
		>5.5173	>6.3919	>6.7057	>6.8134	9
			>16.5520	>19.1756	>20.1171	27
				>49.6560	>57.5269	81
					>148.9689	243

Table 1: Optimal number of initial slots dependent on the dimension n of the identifier space and the known or estimated size $\overline{k}$ of a start set for binary and ternary identifier splitting algorithm

$$p_{\geq 1,i} = 1 - C_i \cdot (1-p)^{n_{idle,i}} \tag{15}$$

The parameter C_i is set to 1 and will be explained later.

Our new medium access control protocol for the RACH can be considered as an unblocking adaptive identifier splitting algorithm. We call it *probing algorithm*. At the beginning of each period it divides the identifier space in a variable number t of consecutive intervals and assigns one slot to each interval. The l-th interval is starting with terminal i_l and ending with terminal $i_{l+1}-1$, with $i_1 = 0$ and $i_t = o^n - 1$. It contains $K_l = i_{l+1} - i_l$ terminals. K_l has to be maximized under the constrain (16).

$$N_l = \sum_{i=i_l}^{i_{l+1}-1} p_{\geq 1,i} < W \tag{16}$$

With the parameter W the probability of a successful transmission can be adjusted. At the end of a period the results of accesses can be used to correct the estimation of $p_{\geq 1,i}$. If no or one transmission happened in a slot, $n_{idle,i}$ is reset to zero and C_i to 1 for all involved station. If a collision occurred in the slot belonging to the l-th interval, the number $N_{coll,l}$ of involved terminals is estimated by (17).

$$N_{coll,l} = N_l \frac{1 - \left(1 - \frac{N_l}{K_l}\right)^{K_l - 1}}{1 - \left(1 - \frac{N_l}{K_l}\right)^{K_l} - N_l \left(1 - \frac{N_l}{K_l}\right)^{K_l - 1}} \tag{17}$$

This estimation is based on the assumption of a binomial distribution of N_l. This is no exact model but a sufficient approximation. We correct the estimation of $p_{\geq 1,i}$ by adjusting C_i:.

$$C = \frac{K_l - N_{coll,l}}{K_l - N_l} \tag{18}$$

$$C_{i,new} = C \cdot C_{i,old} \tag{19}$$

After a successful or no transmission on a slot, C_i of the terminals in the belonging interval is reset to 1.

The approximation made above requires a special treatment of terminals with high $p_{\geq 1,i}$. To avoid high delays, terminals with $p_{\geq 1,i} > W/2$ are polled in specific slots. The same happen with terminals, that have been involved in more than n consecutive collisions with n being the dimension of the identifier space.

The performance of the protocol has been evaluated by stochastic simulations. The number k of terminals, the arrival probability p and the parameter W have been varied. The average and maximum delay as well as the throughput ρ over p for $k = 5$ and $k = 20$ terminals is shown in the diagrams in Fig. 9 (with a relative error $\ll 0.01$). W has been chosen to 1.0 and 1.4. It can be seen, that W has the same effect like the order of a splitting algorithm. Lower values of W lead to shorter delays but reduces the throughput.

The determination of the optimal value of W requires a more precise model of the MAC protocol and the surrounding system. But our results may be a guideline for finding optimal parameter settings of a real MAC protocol.

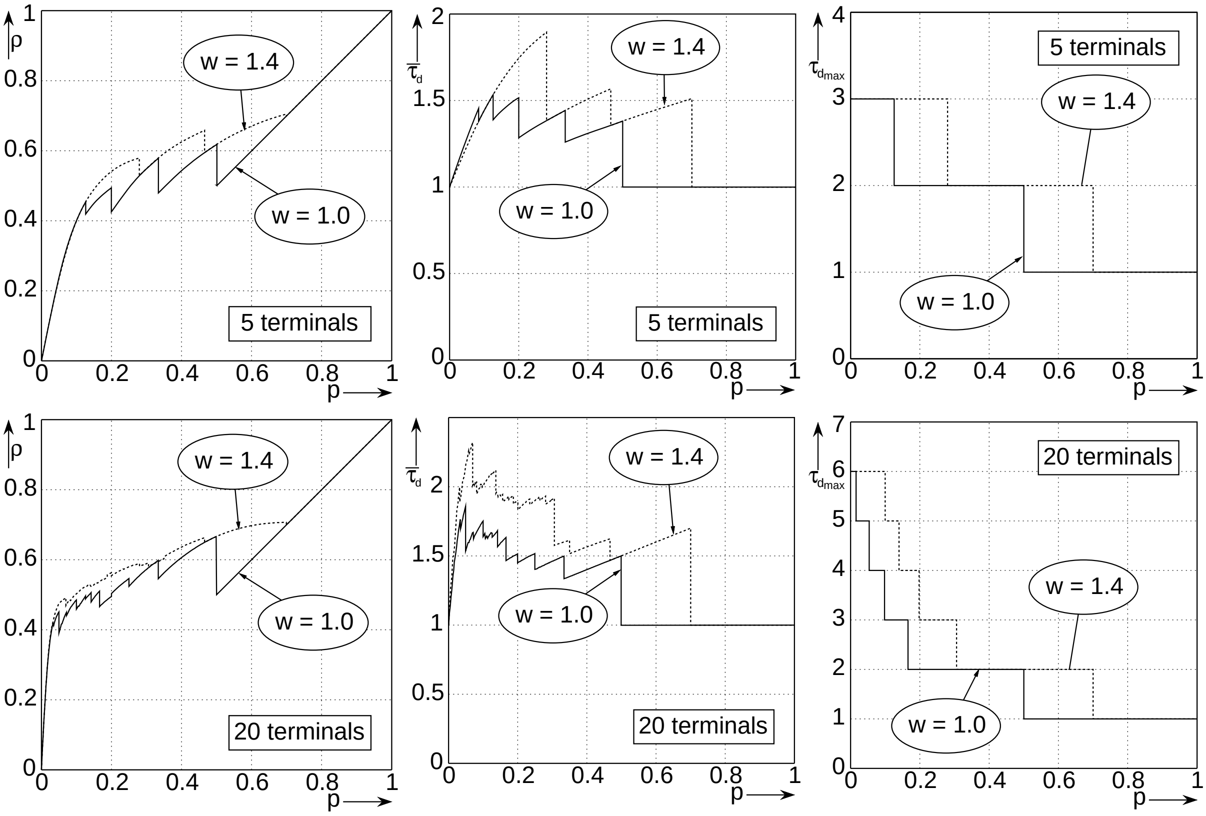

Figure 9: Throughput ρ, average delay $\overline{\tau}_d$ and maximum delay $\tau_{d\,max}$ of probing algorithm with $k = 5$ and $k = 20$ terminals

6 Conclusions

Our new medium access control protocol has been developed taking into account the results of the analysis of the identifier splitting algorithm. The protocol offers a good performance by combining the advantages of the identifier splitting algorithm and pure polling. We assumed an error-free feedback. This is no realistic model for a radio channel with noise and interference. We intent to modify our algorithms in order to use a soft decision feedback. Based on the accuracy of this feedback, the grade of correcting C_l can be adjusted.

7 References

[1] D. Bertsekas, R. Gallager. *Data Networks*. Prentice-Hall, Englewood Cliffs, NJ, 1987.

[2] J. I. Capetanakis. *Tree Algorithms for Packet Broadcast Channels*. IEEE Trans. Inform. Theory, Vol. 25, No. 5, pp. 319–329, 1979.

[3] P. Mathys, P. Flajolet. *Q-ary Collision Resolution Algorithm in Random Access Systems with Free or Blocked Channel Access.* IEEE Trans. Inform. Theory, Vol. 31, pp. 217–243, 1985.

[4] D. Petras, A. Krämling, A. Hettich. *MAC protocol for Wireless ATM: contention free versus contention based transmission of reservation requests.* In *PIMRC'96*, Taipei, Taiwan, October 1996.

[5] ETSI RES10. *HIgh PErformance Radio Local Area Network (HIPERLAN), Requirements and Architectures.* Draft ETR, Sophia Antipolis, France, 1996.