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Stochastic Simulation: A Simplified LRE-Algorithm for Discrete Random Sequences

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Abstract **The LRE-algorithm III (LRE: Limited Relative Error) for the statistical evaluation of discrete random x-sequences yields by simple rules the stationary d.f. $F(x)$ and the so-called *local* correlation function $\varrho(x)$, $-1 \leq \varrho \leq 1$, to be included in a mean quadratic error measure for controlling the simulation run time. The use of this error statement instead of confidence interval statements and the principle of measuring correlation evidence instead of trying to eliminate correlation represent essentials for overcoming the deficiencies of the widespread Batch Means evaluation method.**

The LRE-algorithm III has been verified by test simulations of analytically described queueing systems and recently applied as part of the so-called RESTART/LRE-algorithm for simulating very small probabilities of rare cell loss events in ATM-networks: in a situation being affected by various correlation phenomena the error and run time control procedures of the LRE-method secure the objectivity of numerical simulation results.

Keywords **Stochastic simulation, run length control, LRE-algorithm, discrete random sequences, local correlation, mean quadratic error measure.**

1. Introduction

1.1 Statistical Evaluation and Correlation

Since the early days of applying stochastic simulation in the teletraffic field there has been the major problem how to evaluate random sequences *in the presence of correlation* in order to gain an objective error statement for simulation run length control. Various attempts to adapt common statistical methods for solving this problem were not truly successful, see for instance the deficiencies of the most often used evaluation method *Batch Means* [1], [2], which depends on the elimination of correlation by forming quasi-independent, quasi-normally distributed batch-random variables. In fact, until recently, experts have warned that the evaluation problem could jeopardize the trustworthiness of simulation results in general [2].

To overcome these difficulties the LRE-algorithm II (LRE: Limited Relative Error) has been proposed in [3] as a new statistical evaluation method which takes into account measured evidence of correlation.[1] Its main features are as follows:

a) The LRE-algorithm II determines the d.f. $F(x)$ or compl. d.f. $G(x) = 1 - F(x)$ of a stationary, but otherwise unknown x-sequence by subdividing not the x-axis but the vertical F- resp. G-axis according to prescribed resolution parameters, namely the desired number of intervals k (e.g., $k = 200$) and the lowest value $F_{\min}$ resp. $G_{\min}$ to be established by simulation (e.g., $G_{\min} = 10^{-3}$).

b) The character of the x-sequence might be discrete, continuous, or mixed continuous-discrete: within the limits given by the chosen resolution parameters the LRE-algorithm II is able to detect "rare events" and to establish isolated discrete points as well as discrete points embedded in a continuous surrounding as part of the function $F(x)$ resp. $G(x)$.

c) Instead of trying to eliminate correlation by forming quasi-independent batches or by other manipulations the LRE-algorithm II includes among the objects to be permanently measured the so-called *local correlation coefficient*[2] $\varrho(x)$, $-1 \leq \varrho \leq 1$, which represents a well defined (first order) correlation property of Markovian systems [5], [6], [7] and has been found to be *the* adequate measure for dealing with the correlation problem in simulation [8].

d) In any state of a simulation run the error statement concerning the posterior function[3] $\tilde{F}(x)$ resp. $\tilde{G}(x)$ obtained by evaluating the x-sequence (under observance of strictly controlled large sample conditions) is not expressed by confidence interval methods but by a *mean quadratic error measure*, which depends, among other things, on the measured value[3] $\tilde{\varrho}(x)$ of the local c.c. $\varrho(x)$: the posterior variance $\sigma^2_{\tilde{F}}(x)$ resp. $\sigma^2_{\tilde{G}}(x)$ and, derived from this, the *relative error function* $d_F(x) = \sigma_F(x)/\tilde{F}(x)$ resp. $d_G(x) = \sigma_G(x)/\tilde{G}(x)$.

e) A simulation run is stopped if in the whole prescribed F- resp. G-range of investigation the relative error $d_F(x)$ resp. $d_G(x)$ is below a given error limit $d_{\max}$. To achieve this the sample size n (number of trials) can be steadily increased by producing more simulation output events until the error criterion $d_F(x) \leq d_{\max}$ resp. $d_G(x) \leq d_{\max}$ is fulfilled.

f) The pair of posterior functions $\tilde{F}(x)$, $\sigma^2_{\tilde{F}}(x)$ resp. $\tilde{G}(x)$, $\sigma^2_{\tilde{G}}(x)$ imply also posterior statements concerning the *expectation parameters* (moments) $\mu_\nu = \mathrm{E}\{x^\nu\}$, $\nu = 1, 2, \ldots$ of the x-sequence.

After its introduction in 1988 the LRE-algorithm II has been implemented as part of a C++ simulation system [9], [10][4] and has proved its usefulness in many investigations. The simulation results reported in [11] make evident that this evaluation tool allows for a more deeply understanding of the

[1] A first version of this method [4] was restricted to the evaluation of *independent* random sequences.

[2] Abbreviation: local c.c. $\varrho(x)$.

[3] The notations here differ partially from those in [3] and [8]: $F_n(x) \to \tilde{F}(x)$, $G_n(x) \to \tilde{G}(x)$, $\overline{\rho}_F(x) = \overline{\rho}_G(x) \to \tilde{\varrho}(x)$.

[4] The CNCL [10] simulation library is available via anonymous ftp at ftp.comnets.rwth-aachen.de.

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random behavior of queueing systems, because it provides as a supplement to the d.f. $F(x) = 1 - G(x)$ the function $\varrho(x)$: a relevant quantitative statement about the correlation of the investigated x-sequence.

1.2 Evaluation of Discrete Sequences

Whereas the original LRE-algorithm II [3] has a fairly complicated structure due to its ability to evaluate *general* random sequences (see Item (b) in Section 1.1), the new LRE-algorithm III is distinguished by an elementary structure and by simple operating rules, because it has been designed to evaluate *discrete* random sequences only.

This specialized LRE-version is part of a new simulation system for rare event investigations [12] by combining an interesting type of importance sampling, the so called RESTART-principle [13], [14] with the LRE-evaluation principles: this RESTART/LRE-algorithm allows time saving simulation runs for the determination of extremely low cell loss probabilities in the order of 10^{-9}, which have become an important performance requirement for switching nodes in the future ATM-broadband network [15]. Numerical results obtained by such a method with two or more subsequent simulation phases depend in a sensitive way on a carefully chosen error control procedure.

Due to its basic simplicity the LRE-algorithm III is well suited to display the most important features of the Limited Relative Error-method: the measurement of correlation and the use of a mean quadratic error formula for simulation run length control.

2. The Concept of Local Correlation

We assume now that a discrete r.v. β is generated by the random state sequence of a recurrent $(k+1)$-node Markov chain with transition probabilities[5] $p_{ji} = P(\beta = j|\beta = i)$, whose stationary state probability P_β is uniquely determined by solving the equation system

$$0 = \sum_{i=0}^{k} p_{\beta i} P_i - P_\beta, \quad \beta = 1, 2, \ldots, k,$$

$$P_0 = 1 - \sum_{\beta=1}^{k} P_\beta, \qquad k = 1, 2, \ldots. \tag{1}$$

Then the compl. d.f.[6]

$$G(x) = G_i = \sum_{\beta=i}^{k} P_\beta, \; i - 1 \le x < i, \; i = 1, 2, \ldots, k,$$

$$G_0 = 1,$$
$$G_{k+1} = 0. \tag{2}$$

exists as shown by Fig. 1. Now, at any point x on the real axis, we can split this chain into two parts and define an "$G(x)$-equivalent" 2-node Markov chain, whose transition probabilities $p_0(x)$ and $p_1(x)$ are uniquely given by the equations [8]

$$p_0(x) = \frac{1}{F_i} \sum_{\beta=0}^{i-1} P(\beta) \sum_{j=i}^{k} p_{j\beta},$$

$$p_1(x) = \frac{1}{G_i} \sum_{\beta=i}^{k} P(\beta) \sum_{j=0}^{i-1} p_{j\beta},$$

$$i - 1 \le x < i, \quad i = 1,2, \ldots, k. \tag{3}$$

and determine the local c.c. $\varrho(x)$ of this 2-node chain and the state probability $Q(x)$ of its left state $S_0(x)$ resp. $1 - Q(x)$ of its right state $S_1(x)$

$$\varrho(x) = 1 - [p_0(x) + p_1(x)],$$
$$Q(x) = p_1(x)/[1 - \varrho(x)] \equiv F(x),$$
$$1 - Q(x) = p_0(x)/[1 - \varrho(x)] \equiv G(x),$$
$$i - 1 \le x < i. \tag{4}$$

From this follows the equality

$$p_0(x)F(x) = p_1(x)G(x), \tag{5}$$

and we conclude that the two double sum expressions in eq. (3) are equal.

The function $\varrho(x)$, which has been considered for the first time in 1967 by Blomquist [17], [18], is called to be *local* due to its dependence on the "location" x and in order to distinguish it from the standard *global* correlation coefficient ϱ, which expresses a mean statement with respect to all x-values [5]. This local c.c. $\varrho(x)$ represents a well defined (first order) correlation function of Markovian random processes, which has been analytically derived for the elementary queue M/M/1/N [6] as well as for complex semi-Markov queueing systems [7]. As shown in [5], the function $\varrho(x)$ can also be derived for discrete Markov chains with an *infinite number of states* $k \to \infty$ and in case of *continuous* random variables x like the waiting time in a queue.

3. The LRE-Algorithm III

3.1 Frequency Measurements

We assume now that the compl. d.f. $G(x)$ of a $(k+1)$-node Markov chain with known k-value and known recurrence property, but otherwise unknown values of the transition parameters p_{ji} will be determined by simulation. Therefore it is *a priori* known that a discrete random sequence with a known range of possible values has to be investigated. Due to these elementary prior conditions the "F-Sorting Memory" FS resp. "G-Sorting Memory" GS of the relatively complicated LRE-algorithm II [3] can be completely omitted for operating the simply structured LRE-algorithm III. The only

[5] The notation $p_{ji} = P(j|i)$ instead of the mostly preferred notation $p_{ij} = P(j|i)$ avoids the frequent use of transposed transition matrices and complies with the definition of the stochastic matrix in accordance with standard matrix notation [16].

[6] Here and in the following we use preferably a description by the compl. d.f. $G(x)$ instead of by the d.f. $F(x) = 1 - G(x)$ because in most teletraffic applications there are distributions in the range $0 \le x < \infty$, which have to be investigated with respect to their *right* tail.

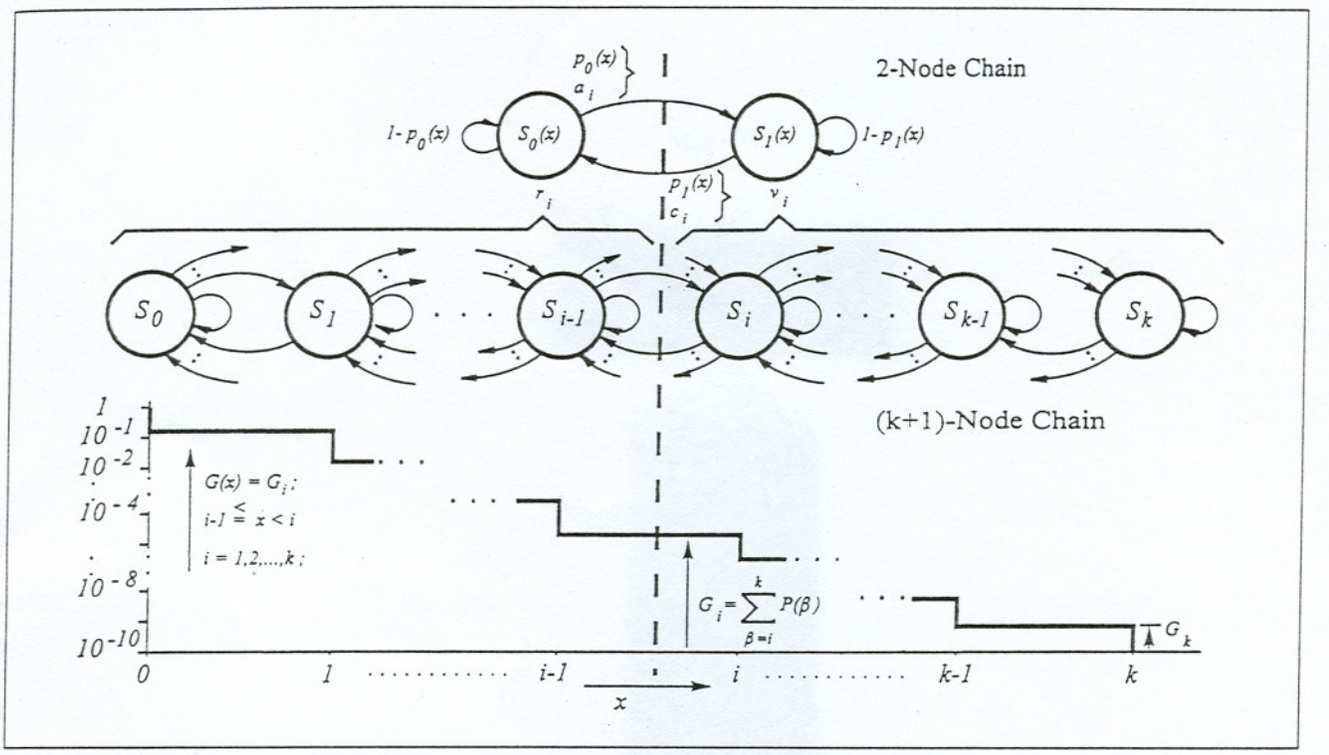

Fig. 1. Discrete $(k+1)$-node Markov chain and the "$G(x)$-equivalent" 2-node Markov chain[6] [8].

memory needed for a G-evaluation is the "G-Result Memory" GR with a record for storing two frequency values for each state i of the $(k+1)$-node chain after this chain has performed n state changes (trials):

a) **The state frequency** h_i how often each state $i = 0, 1, \ldots, k$ has occured in n trials. From this we also obtain the sum frequency

$$v_i = \sum_{\beta=i}^{k} h_\beta, \quad i = 0, 1, \ldots, k, \quad v_0 = n \qquad (6)$$

how often the right state $S_1(x)$ of the 2-node chain has occurred, $i - 1 \le x < i$.

b) **The transition frequency** $c_i, i = 1, 2, \ldots, k$ how often there has been a transition $S_1(x) \to S_0(x)$, that is a transition from any state β in the range $\beta \ge i$ across the separation line at a point x in the interval $i - 1 \le x < i$.

The frequencies v_i and c_i resp. their counterparts $r_i = n - v_i$ and $a_i \approx c_i$ are associated to the "$G(x)$-equivalent" 2-node Markov chain as shown by Fig. 1.

3.2 Statistics of the 2-Node Markov Chain

After the frequencies v_i and c_i have been measured in n trials we now apply the results of two statistical investigations [19], [20] concerning the 2-node Markov chain. If the large sample conditions

$$n \ge 10^3, \quad (r_i, v_i) \ge 10^2,$$
$$(a_i, c_i, r_i - a_i, v_i - c_i) \ge 10 \qquad (7)$$

are fulfilled then we can express the required posterior statements concerning

- the posterior compl. d.f. $\tilde{G}(x)$ and the posterior mean state $\tilde{\beta}$,
- the posterior local c.c. $\tilde{\varrho}(x)$ with correlation factor $cf(x)$ and
- the relative error $d_G(x) = \sigma_G(x)/\tilde{G}(x)$ with respect to the posterior statement $\tilde{G}(x)$,

by the following formulae:

$$\tilde{G}(x) = \tilde{G}_i = v_i/n, \quad \tilde{\beta} = \frac{1}{n}\sum_{i=1}^{k} v_i,$$

$$\tilde{\varrho}(x) = \tilde{\varrho}_i = 1 - \frac{c_i/v_i}{1 - v_i/n},$$

$$cf(x) = cf_i = (1 + \tilde{\varrho}_i)/(1 - \tilde{\varrho}_i),$$

$$d_G(x) = d_i = \left[\frac{1 - v_i/n}{v_i}\, cf_i\right]^{1/2},$$

$$i - 1 \le x < i, \quad i = 1, 2, \ldots, k. \qquad (8)$$

Due to the large sample condition we find — as it should be — that the levels $\tilde{G}_i$ of the posterior function $\tilde{G}(x)$ are simply expressed by the relative frequencies v_i/n. It is the inclusion of the transition-frequency c_i into the evaluation routine that marks the essential progress compared to conventional batch means and confidence interval methods: by this feature we obtain not only the correlation function $\tilde{\varrho}(x)$ but also the (mean quadratic) relative error $d_G(x)$ as a function of $\tilde{\varrho}(x)$ in a *continual* (not batch-subdivided) process

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that allows to increase the number of trials n steadily until the *error condition* $d_i \leq d_{\max}$ is fulfilled, with $d_{\max}$ being the prescribed error limit.

The total needed number of trials n is determined by $d_{\max}$ and by the smallest desired G-value to be established, namely $G_{\min} = \tilde{G}_k$ in the present case; with $d_i = d_k = d_{\max}$ and $cf_i = cf_k$ we obtain from eq. (8)

$$n = \frac{(1 - G_{\min})cf_k}{G_{\min} d_{\max}^2} \approx \frac{cf_k}{G_{\min} d_{\max}^2}, \quad cf_k = \frac{1 + \tilde{\varrho}_k}{1 - \tilde{\varrho}_k}. \tag{9}$$

This formula is useful for estimating simulation run times for given values of $G_{\min}$, $d_{\max}$, and for an estimated value of $\tilde{\varrho}_k$, see Fig. 6 [3]. Assuming for instance $G_{\min} = 10^{-4}$, $d_{\max} = 0.05$ or 5 % and $\tilde{\varrho}_k = 0.8$ (correlation factor $cf_k = 9$), we find that about $n = 3.6 \times 10^7$ trials are needed.[7]

3.3 The Procedure of LRE-Algorithm III

We assume that the parameters of the simulation $d_{\max}$ and $G_{\min}$ have been prescribed. Then the LRE-algorithm III evaluates the β-sequence $\beta_1, \beta_2, \ldots, \beta_n$ produced by a $(k+1)$-node Markov chain or by any other discrete random generator and augments the number of events n as far as necessary until all levels $\tilde{G}_i \geq G_{\min}$ of the posterior d.f. $\tilde{G}(x)$ have been established with a rel. error $d_i \leq d_{\max}$. After the contents of the "G-Result Memory" GR have been set to zero the main procedure steps $(\mathrm{St}_0), \ldots, (\mathrm{St}_3)$ can be outlined as follows.

(St_0) Initiation:
Set $n := 0$; $i_S := 0$;
Reset the β-generator to state S_0;
Receive $\beta_0 = 0$.

(St_1) Input of M-Block:
Set $\nu := n$;
While $n < \nu + M$ Do Set $n := n + 1$;
Receive β_n;
Set $i := \beta_n$; $h_i := h_i + 1$;
If $\beta_{n-1} > i$ Then Set $\lambda := \beta_{n-1}$;
Set $c_{i+1} = c_{i+1} + 1, \ldots, c_\lambda = c_\lambda + 1$.

(St_2) Run Time Control:
Set $v_0 := n$; $i := i_S + 1$;
(St_2^*) While $i < k$ Do
Set $v_i := v_{i-1} - h_i$; $r_i := n - v_i$; $a_i := c_i$;
If "one of the conditions eq. (7) is *not* fulfilled"
Then Goto (St_1)
Else Compute by eq. (8) $\tilde{\varrho}_i$ and rel. error d_i;
If $d_i > d_{\max}$ Then Goto (St_1)
Else Set $i_S := i$; $i := i + 1$; Goto(St_2^*).

(St_3) Termination:
Compute for $i = 1, 2, \ldots, k$ by eq. (6) the final sum frequencies v_i and by eq. (8) the final posterior values $\tilde{G}_i$ and $\tilde{\varrho}_i$; Plot the step functions $\tilde{G}(x)$ and $\tilde{\varrho}(x)$.

[7] This number of trials should be no problem for simulation on present day computers. But lowering the smallest G-value to $G_{\min} = 10^{-9}$ would raise the number of trials to the excessive value $n = 3.6 \times 10^{12}$: obviously, this would be a situation, where the "straight-forward" LRE-simulation has to be replaced by rare event simulation techniques [13], [12].

Comments.

(St_0) Initiation: other, more complicated initiation rules are conceivable; if for instance the β-sequence is the occupancy of a queueing system G/G/1/N with buffer size $N = k - 1$ and the value of interest is the loss probability, it is recommended to start by a reset to the "idle period" with $\beta_0 = 0$ and to relate the sequence $\beta_1, \beta_2, \ldots, \beta_n$ to the instants "immediately prior to arrivals".

(St_1) Input of M-Block: by means of this feature the control procedure (St_2) has to be performed only after a certain progress of the simulation process: the evaluation of M further β-values. The block size M is typically chosen in the range $M = 10^2 \ldots 10^3 \ldots 10^4$.

(St_2) Run Time Control: the separation mark i_S separates all states $i = 1, 2, \ldots, i_S$, which have already fulfilled the control conditions, from the other states to the right $i = i_S + 1, i_S + 2, \ldots, k$, which have not yet passed the control.

4. Simulation Results

The elementary finite buffer queue M/M/1/N has recently been investigated in [6] with respect to the function pair $G(x), \varrho(x)$ of its occupancy $\beta = 0, 1, \ldots, k$ with $k = N+1$ being the capacity of a single server queue. Adapting eqs. (7) and (12) [6] to the notation of the present paper and assuming a traffic load $\eta < 1$ these functions are expressed by the formulae:

$$G(x) = G_i = \frac{\eta^i - \eta^{k+1}}{1 - \eta^{k+1}},$$

$$\varrho(x) = \varrho_i = 1 - \frac{(1-\eta)(1-\eta^{k+1})\eta^i}{(1+\eta)(1-\eta^i)(\eta^i - \eta^{k+1})},$$

$$i - 1 \leq x < i, \quad i = 1, 2, \ldots, k. \tag{10}$$

A simulation run with evaluation by LRE-algorithm III has yielded the corresponding posterior functions $\tilde{G}(x)$, $\tilde{\varrho}(x)$ shown by Fig. 2: within the tolerances given by the chosen error limit $d_{\max} = 0.05$ these two simulated step functions are in full agreement with eq. (10) and with the analytically derived diagrams of Fig. 2 [6]. Only the local c.c. $\tilde{\varrho}(x)$ shows some deviations[8] at the right end for $x = 24$ and $x = 25$, which are acceptable considering the very low probabilities for these states. These results can be improved using the RESTART/LRE method [12].

Using the values $G_{\min} \approx 1.1 \times 10^{-6}$ and $\rho_k \approx 0.4$ the number of trials can also be calculated from eq. (9) to be about 8.4×10^8. Assuming a realistic rate of about 10^4 trials/s for a simple model this leads to a simulation run time of about one full day (23.3 h).

Fig. 3 shows the simulated posterior functions $\tilde{G}(x)$ and $\tilde{\varrho}(x)$ of the finite buffer queue SSMP/D/1/N with 'SSMP' being a 'Special Semi Markov Process' for describing a correlated interarrival time in ATM-networks [21] and with 'D' denoting a constant service time according to the constant ATM-cell length. This complex queueing system has been analytically investigated [7].

[8] The theoretical curve is symmetrical with $\varrho_1 = \varrho_k$.

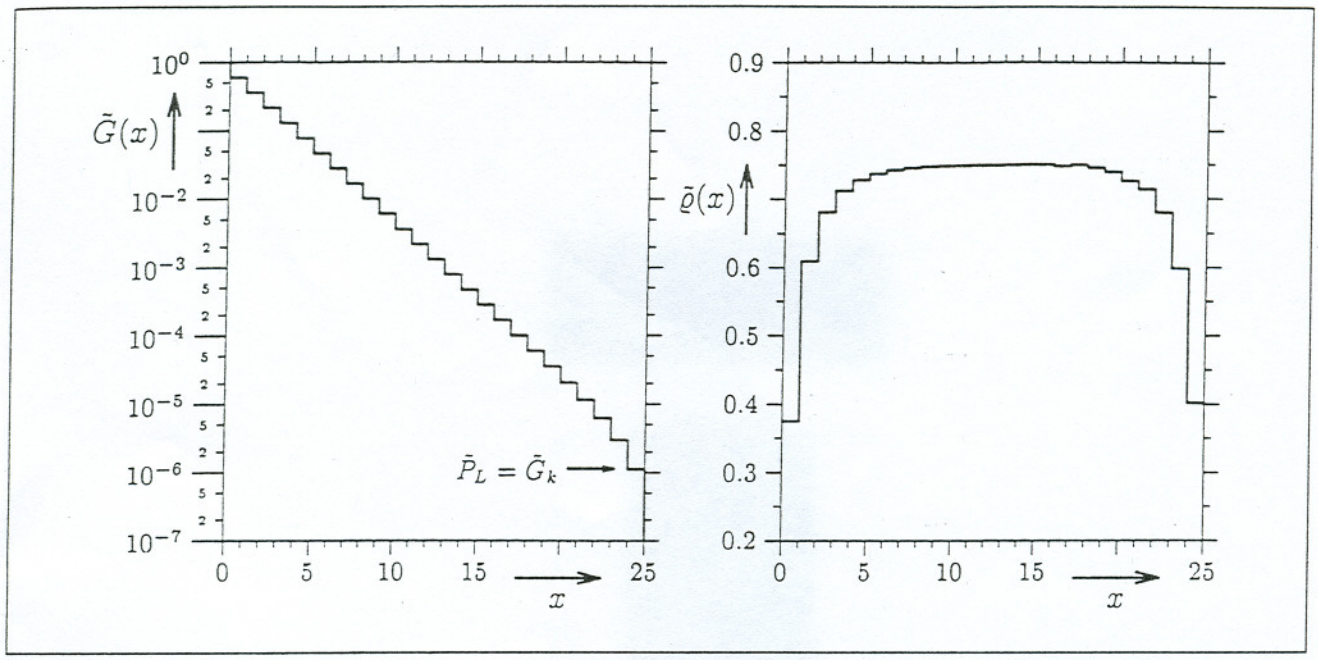

Fig. 2. LRE simulation of queue M/M/1/N: compl. d.f. $\tilde{G}(x)$ and local c.c. $\tilde{\varrho}(x)$, $i-1 \leq x < i$ of occupancy $\beta = i$. Buffer size: $N = k-1 = 24$. Traffic load: $\eta = 0.6$. Loss probability: $P_L = G_k \approx 1.1 \times 10^{-6}$. Simulation parameters: $d_{\max} = 0.05$, $G_{\min} = 10^{-6}$. Number of needed trials: $n \approx 8.4 \times 10^8$.

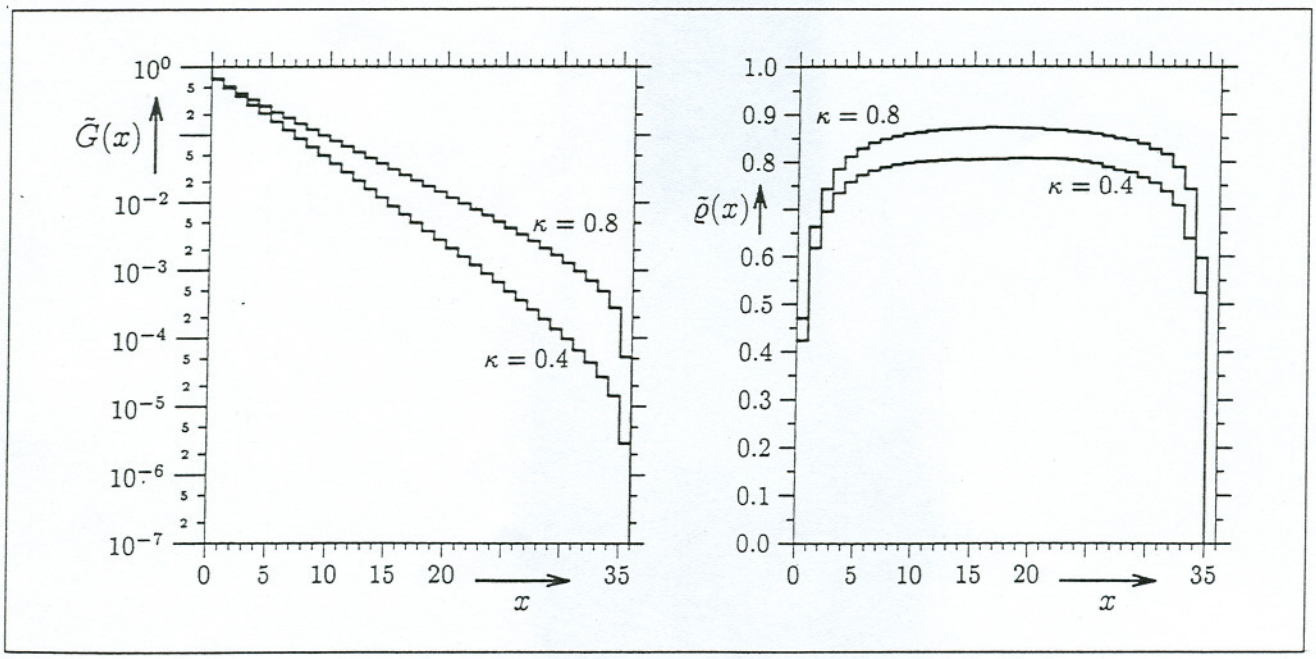

Fig. 3. LRE simulation of queue SSMP/D/1/N: compl. d.f. $\tilde{G}(x)$ and local c.c. $\tilde{\varrho}(x)$, $i-1 \leq x < i$ of occupancy $\beta = i$. Buffer size: $N = k-1 = 35$. Service time: $D = 2$. Traffic load: $\eta = 0.7$. Correlation coefficient of interarrival time: $\kappa = 0.4$ resp. $\kappa = 0.8$. Loss probability: $P_L = G_k \approx 5 \times 10^{-6}$ resp. $\approx 10^{-4}$. Simulation parameters: $d_{\max} = 0.05$, $G_{\min} = 10^{-6}$. Number of trials: $n \approx 78 \times 10^6$ resp. $n \approx 4.5 \times 10^6$.

A comparison between the two diagrams Fig. 4 and Fig. 5 in [7] and Fig. 3 shows clearly that the LRE-algorithm III traces, also in case of this complicated simulation object and within the tolerances given by $d_{\max} = 0.05$ and the parameter values from [7], the step functions $\tilde{G}(x)$ and $\tilde{\varrho}(x)$ in good agreement with theoretical results. Only for small

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values of x ($x = 0, 1, 2$) significant differences for $\tilde{\varrho}(x)$ can be detected. These differences need further investigation. The number of trials used in the simulation are in the order of the theoretical values given in eq. (9). The other parameter values given in [7] can only be verified with the combined RESTART/LRE method [12] as $G_{\min}$ is in the order of 10^{-9} in these examples.

5. Concluding Remarks

1) Due to its elementary structure the LRE-algorithm III exhibits in simple terms the "Limited Relative Error"-principles, namely how to gain from simulated data the local c.c. $\varrho(x)$ and how to derive therefrom a "correlation dependent" error measure for controlling the simulation run time.
2) The experience with a great number of simulative investigations indicates that the hitherto prevailing evaluation difficulties with correlated x-sequences can be eliminated by including the function $\varrho(x)$ into the evaluation procedure.
3) The local c.c. $\varrho(x)$ represents a relevant correlation statement supplementing the distribution statement $F(x) = 1 - G(x)$.

   Therefore, the mathematical analysis of the correlation function $\varrho(x)$ resp. of its more general form $\varrho_\kappa(x)$ of order $\kappa = 1, 2, \ldots, \varrho_1(x) \equiv \varrho(x)$ deserves to become an object of interest within the theory of random processes and queueing systems.
4) The perfect agreement between LRE-simulated results (as reported in Section 4 and in earlier publications) and analytically derived results represents a "mutual verification" of these results and gives rise to the following consideration:

   The introduction of the function $\varrho(x)$ into the fields of statistics and simulation on the one side and into the fields of probability and analytical methods on the other side helps to clarify the basic interdependencies and the common ground of these fields.
5) The main application of LRE-algorithm III refers to rare event simulations by means of importance sampling. The RESTART/LRE-algorithm with two simulation phases [12] has proved that rare event probabilities in the order of 10^{-9} can be reliably simulated. The extension of this algorithm to $m = 2, 3, \ldots$ phases would be of considerable interest, because — under condition of proper Markovian state-dependencies — it would allow the simulation of extremely rare event probabilities of e.g., 10^{-18}, see the extension of the RESTART-method in [22].

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