Time Efficient Simulation: The LRE-Algorithm for Producing Empirical Distribution Functions with Limited Relative Error*

by Friedrich Schreiber **

Report from the Lehrstuhl für Allgemeine Elektrotechnik und Datenfernverarbeitung, Technische Hochschule Aachen

Considering the problem "measurement of an unknown distribution function $F(x)$" it is shown that recently derived formulae for the so called objective empirical distribution function $F_n(x)$ and its relative error $d_F(x)$ can be made accessible to practical application by implementing the LRE-algorithm ("Limited Relative Error"). This algorithm controls the number of independent trials n and the sorting of measured random data, such that $F_n(x)$ is traced down to a desired resolution (minimum value) $F_{\min}$, whereby the relative error $d_F(x)$ is limited below a given maximum value $d_{F\max}$ and the computertime for sorting is reduced effectively.

Simulation examples demonstrate that the LRE-algorithm can be applied to all stationary processes with independent random variables of continuous and/or discrete nature and that it is well suited to detect rare event details of $F(x)$ within the given resolution $F_{\min}$.

Zeiteffiziente Simulation: der LRE-Algorithmus zur Erzeugung empirischer Verteilungsfunktionen mit begrenztem relativem Fehler

Es wird das Problem „Messung einer unbekannten Verteilungsfunktion $F(x)$" erneut aufgegriffen und gezeigt, daß kürzlich abgeleitete Formeln für die sogenannte objektive Verteilungsfunktion $F_n(x)$ und den zugeordneten relativen Fehler $d_F(x)$ durch den LRE-Algorithmus („Limited Relative Error") der praktischen Anwendung zugänglich gemacht werden können. Dieser Algorithmus steuert die Anzahl unabhängiger Versuche n und die Sortierung der gemessenen Zufallswerte so, daß $F_n(x)$ bis herab zu einer gewünschten Auflösung (Minimalwert) $F_{\min}$ dargestellt wird, wobei der relative Fehler $d_F(x)$ unterhalb eines vorgegebenen Maximalwertes $d_{F\max}$ begrenzt bleibt und die Rechnerzeit für den Sortiervorgang wirksam reduziert wird.

Einige Simulationsbeispiele machen deutlich, daß der LRE-Algorithmus bei allen stationären Prozessen mit unabhängigen Zufallsvariablen vom kontinuierlichen und/oder diskreten Typ anwendbar ist und daß er geeignet ist, innerhalb der gegebenen Auflösung $F_{\min}$ Details der Funktion $F(x)$ im Bereich seltener Ereignisse aufzudecken.

1. Introduction [1]

Following the invention of the early traffic machines simulation techniques on digital computers have become an indispensable tool to investigate complex teletraffic systems [1]. One of the most important tasks in simulation is to determine approximately the a priori unknown d.f. $F(x)$ of a r.v. x. Often this is achieved by some kind of *frequency distribution method*, where due to a relatively large number of trials n the frequency "k_u events in interval u" of m intervals ($u = 1, 2, \ldots, m$) is assumed to be approximately normally distributed. This then leads to the application of conventional t-distribution confidence intervals for the control of error and simulation run length [2].

In [3] it has been shown that confidence intervals belong to the subjective methods in statistics and can be favorably replaced by easy to handle objective error formulae derived from the so called *objective Bayes-statistics*. In the mean time the simulation tool has been used to put the fundamentals of these statistics (including an "extended Bayes-postulate") on an experimental basis [4]. Thus simulation techniques on computers help to gain improved statistical methods and these methods on the other hand improve simulation techniques.

2. The Objective Empirical Distribution Function

2.1. Frequency distribution methods suffer from the disadvantage that the information with respect to the individual magnitude of all x-values collected in a certain interval is lost. Such information loss can be avoided by the following solution to the problem "measurement of an unknown distribution function".

Given a r.v. x with unknown d.f. $F(x)$ in the unknown range $x_{\mathrm{I}} \leqq x \leqq x_{\mathrm{II}}$ the x-values measured in n *independent* trials can be sorted to yield the

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** Prof. Dr.-Ing. F. Schreiber, Lehrstuhl für Allgemeine Elektrotechnik und Datenfernverarbeitung, Technische Hochschule, Templergraben 55, D-5100 Aachen.

[1] Abbreviations: r.v. ≙ random variable, p.f. ≙ probability function, p.d.f. ≙ probability density function, d.f. ≙ distribution function, c.d.f. ≙ complementary distribution function. In order to simplify notation the same symbol is used for a r.v. and its value.

ordered vector

$$(x_r) = (x_1, x_2, \ldots, x_n)\,; \qquad x_r \leqq x_{r+1}\,, \quad r = 1, 2, \ldots, n-1\,. \tag{1}$$

Then for any given point x on the real axis we observe r values "left of x" in the range $\leqq x$ and $v = n - r$ values "right of x" in the range $> x$. The evaluation of these observations leads to the so called objective empirical distribution functions $F_n(x)$ and $G_n(x)$ expressing the objective posterior approximations to $F(x)$ resp. $G(x) = 1 - F(x)$; in addition we obtain error measures to judge these approximations. The following formulae have been derived in [5] and are presented here in a form which covers the generalized case that the unknown d.f. $F(x)$ may contain continuous sections and steps at discrete points [6]:

Objective empirical distribution function

$$\left.\begin{aligned} F_n(x) &= (r+1)/(n+2)\,, \\ G_n(x) &= 1 - F_n(x) = (v+1)/(n+2) \end{aligned}\right\} n \geqq 1\,, \tag{2a, b}$$

Absolute error (standard deviation)

$$\sigma_F(x) = \sigma_G(x) \equiv \sigma(x) = \frac{1}{n+2}\left[\frac{(r+1)(v+1)}{n+3}\right]^{1/2} \tag{3}$$

Relative error (coefficient of variation)

$$\left.\begin{aligned} d_F(x) &= \sigma(x)/F_n(x) = \left[\frac{n-r+1}{(n+3)(r+1)}\right]^{1/2}, \\ d_G(x) &= \sigma(x)/G_n(x) = \left[\frac{n-v+1}{(n+3)(v+1)}\right]^{1/2} \end{aligned}\right\} \tag{4a, b}$$

$$n \geqq 2 \left\{\begin{aligned} &x_r \leqq x \leqq x_{r+1} \\ &r = -1, 0, 1, \ldots, n, n+1 \\ &\left.\begin{aligned} x_{-1} &= -\infty \\ x_0 &= x_{\mathrm{I}} \\ x_{n+1} &= x_{\mathrm{II}} \\ x_{n+2} &= \infty \end{aligned}\right\} \\ &v = n - r \end{aligned}\right.$$

These formulae depend on the objective Bayes-statistics applied to the case of a binomial random process, see e.g. formula table in [3]. The step functions $F_n(x)$ and $G_n(x)$ have in case of a purely continuous r.v. x a step size $1/(n+2)$ and are closely related to but not identical with the empirical distribution function known from literature having a step size $1/n$, see e.g. [7].

2.2. The straightforward application of the objective formulae eqs. (2a, b) and (4a, b) gives rise to a problem which can be introduced by the simulation example Fig. 1 depicting $G_n(x)$ eq. (2b) and $d_G(x)$ eq. (4b) in case of an E_5-distributed r.v. x (Erlang-distribution of order $k = 5$). We find that the measured function $G_n(x)$ has an unbalanced relative error curve $d_G(x)$ leading from a region of low relative error to a region of high relative error at the right tail of $G_n(x)$. If we would be interested in $F_n(x) = 1 - G_n(x)$ instead of $G_n(x)$ then due to eq. (4a) the high and low relative error regions would change sides, see e.g. Fig. 7 in [5]. Obviously the straightforward application of eqs. (2a, b) and (4a, b) results *principally* in unbalanced relative error curves.

2.3. Let us assume that the schedule of a simulation run contains two predescribed parameters:

a) the "resolution" $F_{\min}$ resp. $G_{\min}$ i.e. the lowest value to be determined of the empirical d.f. $F_n(x)$ resp. c.d.f. $G_n(x)$;
b) the maximum admissible value $d_{F\max}$ of the relative error $d_F(x)$ within the range $F_{\min} \leqq F_n(x) \leqq 1$ resp. $d_{G\max}$ of $d_G(x)$ within the range $1 \geqq G_n(x) \geqq G_{\min}$.

From eqs. (2a, b) and (4a, b) it is easily derived that these two parameters determine the number of trials $n_{\min}$ which are at least necessary for the simulation run

$$n_{\min} = \frac{1 - F_{\min}}{F_{\min}\, d^2_{F\max}} - 3\,. \tag{5a, b}^{2}$$

The diagram Fig. 2 expresses the well known fact that the low error simulation of the "rare event"-tail region of a distribution function with $F_{\min}, G_{\min} \ll 1$ is expensive with respect to a great number of trials.

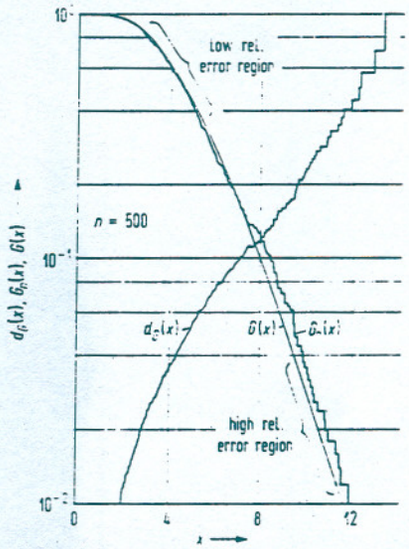

Fig. 1. E_5-distribution: objective empirical c.d.f. $G_n(x)$ eq. (2b) and relative error $d_G(x)$ eq. (4b) ("straightforward sorting").

[2] In this and other formulae eq. (...a) is associated to the d.f. $F_n(x)$. The corresponding eq. (...b) associated to the c.d.f. $G_n(x)$ is given by replacing as far as applicable: $F_{\min} \to G_{\min}$, $d_{F\max} \to d_{G\max}$, $F_n(x) \to G_n(x)$, $d_F(x) \to d_G(x)$, $r \to v$.

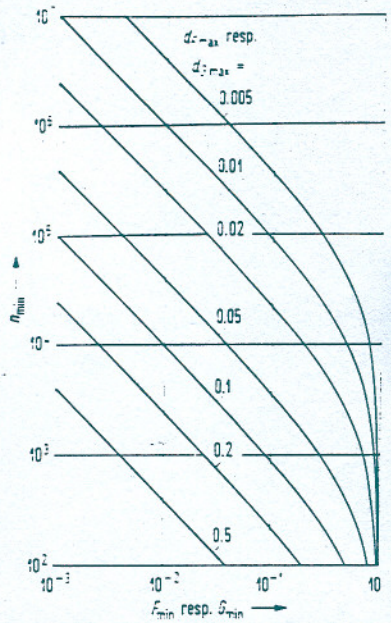

Fig. 2. Number of trials n_{min} eqs. (5a, b), which are at least necessary to fulfill the requirements 2.3a) and b). Curve parameter: maximum relative error $d_{F\max}$ resp. $d_{G\max}$.

If we assume e.g. $F_{min} = 10^{-3}$ and $d_{F\max} = 10^{-1}$ then due to Fig. 2 we must execute $n_{min} \approx 10^5$ trials: this is unavoidable. But we must also sort this number of measured x-values in order to obtain the ordered vector (x_r) eq. (1) and this would cause a prohibitively high consumption of computertime for a sorting effort which yields an unneeded extremely low relative error $d_F(x)$ in a large part of the empirical distribution function.

3. The LRE-Algorithm [3]

3.1. The desired algorithm for handling the evaluation of simulated data should yield a *balanced* relative error curve $d_F(x)$ within a given region of $F_n(x)$ and should also allow an essential reduction of computertime for sorting. For this purpose we combine eqs. (2a, b) with eqs. (4a, b) eliminating n and express the relative error in the form

$$d_F(x) = d_{F\max}\left[\frac{1-F_n(x)}{1+d_{F\max}^2 \cdot F_n(x)}\right]^{1/2}, \quad d_{F\max}^2 = \frac{1}{r+1}, \tag{6a, b}$$

see Fig. 3. From this equation we conclude that in order to stay below a given maximum relative error $d_{F\max}$ resp. $d_{G\max}$ we may at any point x of the simulated curve $F_n(x)$ resp. $G_n(x)$ limit the number r resp. v of measured x-values in the range $\leqq x$ resp. $> x$ according to the formula

$$r = \text{ENTIER}\,(d_{F\max}^{-2}) - 1\,,$$
$$v = \text{ENTIER}\,(d_{G\max}^{-2}) - 1\,.$$

We also conclude from eqs. (6a, b) that if we apply an algorithm keeping r resp. v constant then as

[3] LRE ≙ Limited Relative Error.

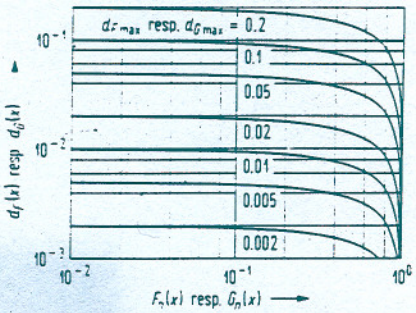

Fig. 3. Relative error $d_F(x)$ resp. $d_G(x)$ eqs. (6a, b). Curve parameter: maximum relative error $d_{F\max}$ resp. $d_{G\max}$.

shown in Fig. 3 we have in the range $F_n(x) < 10^{-1}$ resp. $G_n(x) < 10^{-1}$ (which is of predominant interest in simulation) a practically constant relative error $d_F(x) \lessapprox d_{F\max}$ resp. $d_G(x) \lessapprox d_{G\max}$. This then is in effect the desired balanced error curve.

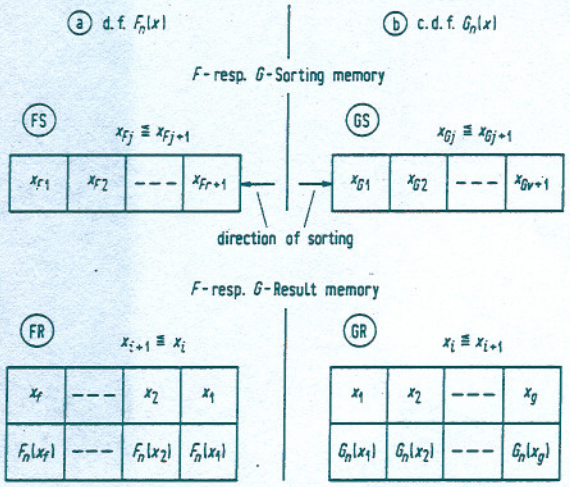

Fig. 4. Structure of memories to be implemented in the evaluation program for the LRE-algorithm, see Section 3.2.

3.2. As shown in Fig. 4 the evaluation program for performing the LRE-algorithm proposed here must provide a F-Sorting memory FS resp. G-Sorting memory GS and a F-Result memory FR resp. G-Result memory GR. The i-th result x_i $(i = 1, 2, \ldots)$ stored in FR resp. GR is obtained after a number of trials n_i has been executed which is unknown in advance. This indicates the main feature of the balanced error-algorithm: instead of having $n = \text{const}$ and r resp. v a measured variable as in Fig. 1 according to the straightforward use of eqs. (2a, b) and (4a, b) we have now $r = \text{const}$ resp. $v = \text{const}$ eqs. (7a, b) whereas the number of trials $n = n_i$ is a measured variable. We will describe the algorithm now step by step for the general case that $F_n(x)$ and/or $G_n(x)$ have to be determined.

3.2.1. Initial phase

a) Given the desired maximum relative error $d_{F\max}$ resp. $d_{G\max}$ compute the integers r resp. v eqs. (7a, b).

b) Execute $n_1 = r+1$ resp. $n_1 = v+1$ trials, sort the produced random x-values into FS resp. GS yielding the initial ordered vector

$(x_{F1}, x_{F2}, \ldots, x_{Fr+1})$, $x_{Fj} \leqq x_{Fj+1}$, $j = 1, 2, \ldots, r$

resp.

$(x_{G1}, x_{G2}, \ldots, x_{Gv+1})$, $x_{Gj} \leqq x_{Gj+1}$, $j = 1, 2, \ldots, v$.

c) Store the first result $x_1 = x_{Fr+1}$ and $F_n(x_1)$ eq. (2a) in FR resp. $x_1 = x_{G1}$ and $G_n(x_1)$ eq. (2b) in GR.[4]

3.2.2. Recursive phase

a) After the i-th result x_i, at a total number of n_i trials ($i = 1, 2, \ldots$), has been obtained execute as many further trials n_i+1, $n_i+2, \ldots$ until, at a certain number of trials $n_{i+1} > n_i$, the generated x-value fulfills for the first time the relation $x \leqq x_{Fr+1}$ resp. $x \geqq x_{G1}$.

b) Sort x into FS resp. GS thereby eliminating the previous largest value x_{Fr+1} resp. smallest value x_{G1}.

c) Store the $(i+1)$-st result $x_{i+1} = x_{Fr+1}$ and $F_n(x_{i+1})$ eq. (2a) in FR resp. $x_{i+1} = x_{G1}$ and $G_n(x_{i+1})$ eq. (2b) in GR.[4]

d) Assuming the lowest value $F_{\min}$ resp. $G_{\min}$ according to 2.3a) to be known check the relation $F_n(x_{i+1}) < F_{\min}$ resp. $G_n(x_{i+1}) < G_{\min}$:

- if this relation is false set $i := i+1$ and go back to 3.2.2a);
- if it is true stop simulation and continue with 3.2.3. The final number of measured values stored in FR resp. GR will be

  f resp. g;

  the final number of trials

  n_F resp. n_G

  is in practice slightly above the value $n_{\min}$ computed by eqs. (5a, b).

3.2.3. Output phase

The contents of FR resp. GR determine the plot of the empirical d.f. resp. c.d.f.

$$F_n(x) = F_n(x_i), \quad x_i \leqq x \leqq x_{i-1}, \quad i = 1, 2, \ldots, f; \; x_0 = x_{II},$$
$$G_n(x) = G_n(x_i), \quad x_{i-1} \leqq x \leqq x_i, \quad i = 1, 2, \ldots, g; \; x_0 = x_I \qquad (8a, b)$$

where the nonmeasured points x_0 are assumed to be unknown, see Section 2.1.

Remarks

a) With the LRE-algorithm the objective statements on basis of eqs. (2a, b) are restricted to the points x_i stored in FR resp. GR, because only for these points the integer r resp. v *and* the number of trials n_i are known quantities. Between two consecutive points x_i, x_{i+1} we are principally free to interpolate in accordance with the fundamental character of a distribution function. But taking into account the possibility that the a priori unknown d.f. $F(x) = 1 - G(x)$ may contain steps it is advisable to plot at first the above *conservative* empirical step functions eqs. (8a, b) and to inspect these functions carefully before any extra- and interpolation takes place, see section 4 in [6].

b) By introducing eqs. (8a, b) in eqs. (6a, b) the relative error $d_F(x)$ resp. $d_G(x)$ can be plotted for control purposes. In practical applications this is not necessary because the error is guaranteed to be below the given maximum value $d_{F\max}$ resp. $d_{G\max}$, see Section 3.1.

[4] Eqs. (2a, b) have to be used here with $n = n_i$ and r = const resp. v = const according to eqs. (7a, b).

4. Measurement Results

The LRE-algorithm is demonstrated by three examples. In all cases the same parameter values were chosen: $d_{F\max}$, $d_{G\max} = 10^{-1}$; $F_{\min}$, $G_{\min} = 10^{-3}$, see Section 2.3. Due to eqs. (5a, b) this leads inevitably to a final number of trials in the order of magnitude $n_F, n_G \approx 10^5$.

In order to judge the quality of the approximations the ideal function $F(x)$ resp. $G(x)$ has been added to each diagram; in normal practice of course this comparison cannot be made because $F(x) = 1 - G(x)$ is unknown.

4.1. In Fig. 5 the LRE-algorithm has been applied to the E_5-distribution. Compared to the diagram Fig. 1 obtained by the straightforward application of eq. (2b) we note in Fig. 5 that the logarithmic step sizes of $G_n(x)$ do not increase toward the tail of the curve. This is a typical feature of all LRE-diagrams with logarithmic ordinate.

4.2. The measurement Fig. 6 shows an example where the LRE-algorithm has traced a generalized c.d.f. $G_n(x)$ with continuous sections and steps,

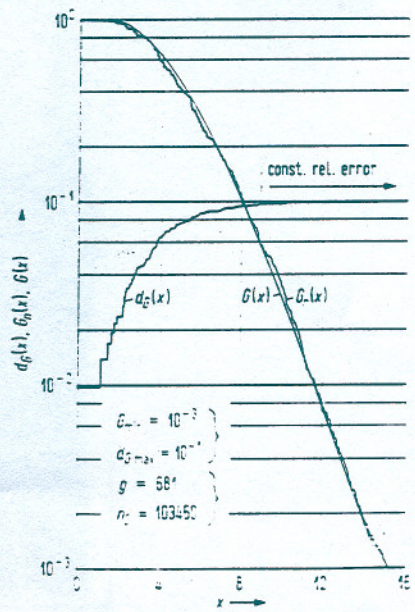

Fig. 5. E_5-distribution: empirical c.d.f. $G_n(x)$ eq. (8b) and associated relative error $d_G(x)$ eq. (6b) gained by the LRE-algorithm. compare Fig. 1.

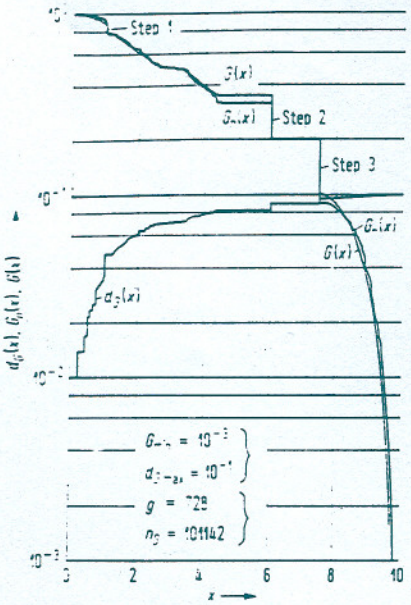

Fig. 6. Random process with four cos-sections and three steps as defined in Appendix A.1 [6]: empirical c.d.f. $G_n(x)$ eq. (8b) and associated error $d_G(x)$ eq. (6b).

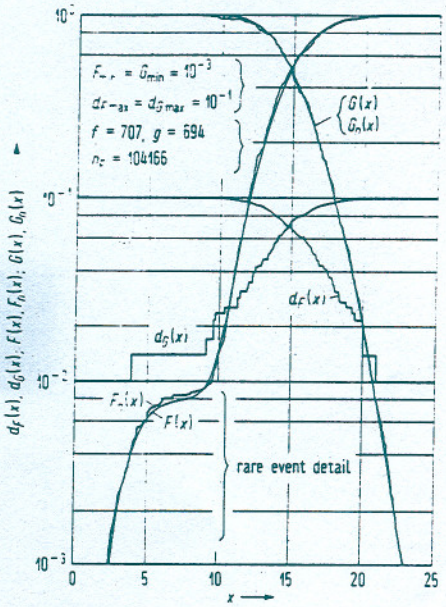

Fig. 7. Mixed E_8-, E_{45}-distribution (see Appendix): empirical d.f. $F_n(x)$ and c.d.f. $G_n(x)$ eqs. (8a, b) and associated relative error $d_F(x)$ and $d_G(x)$ eqs. (6a, b). The rare event detail has been detected by the LRE-algorithm as part of the $F_n(x)$-curve.

compare the straightforward evaluation of the same random process in [6].

4.3. The measurement Fig. 7 is of special interest because it demonstrates that the LRE-algorithm is well suited to detect rare event details of a random process: in this example a mixed E_8-, E_{45}-distribution with a two peak p.d.f. $f(x)$ where the first peak is very weak compared to the second, see Fig. 9. Indeed once the resolution level F_{min} resp. G_{min} (and the maximum relative error) have been set then the LRE-algorithm will trace automatically all details within this level. Theoretically rare event details could also be detected by the straightforward application of eqs. (2a, b) but this would fail in practice because of the excessive expense for sorting, see Section 5.

In Fig. 7 the rare event detail is displayed within the left tail of the d.f. $F_n(x)$ and cannot be detected within the c.d.f. $G_n(x)$. From this we conclude that the rare event analysis of *both* tails of a truly unknown distribution function requires the application of the LRE-algorithm for both empirical functions: the d.f. $F_n(x)$ *and* the c.d.f. $G_n(x)$. Using all memory devices in Fig. 4 simultaneously this can be achieved in a single evaluation run.

We also conclude that the relative error $d_F(x)$ resp. $d_G(x)$ eqs. (4a, b) and not the absolute error $\sigma(x)$ eq. (3) is relevant for the control of the evaluation.

5. Reduced Expense for Sorting

The total computertime for performing a certain simulation resp. statistical evaluation program will contain a certain amount T_{sort} for sorting the measured random x-values. If the speed of a computer is roughly characterized by its average number of operations per time unit ("operation intensity") μ_{op}[MOp/s], then the normalized sorting time $\mu_{op} T_{sort}$[MOp] is the average number of operations for performing a specified sorting job and represents a sorting expense measure which is to some extent independent of a special computer. Based on measurement of T_{sort} the quantity $\mu_{op} T_{sort}$ has been depicted in Fig. 8 as a function of $d_{F\,max}$ resp. $d_{G\,max}$

a) for straightforward sorting of *all* measured x-values as in Fig. 1,
b) for selected sorting according to the LRE-algorithm as in Fig. 5 to Fig. 7.

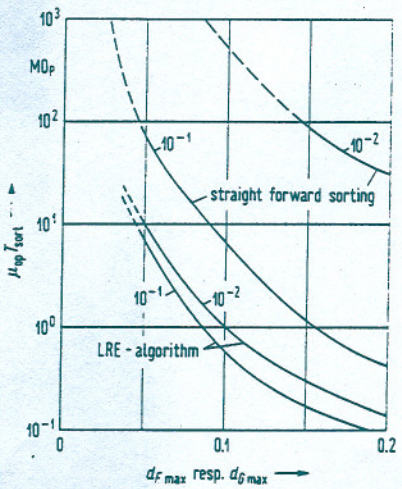

Fig. 8. Typical sorting times T_{sort} measured on a computer Siemens 4004/151 with $\mu_{op} = 0.25$ MOp/s. Curve parameter: resolution F_{min} resp. G_{min}.

Using eqs. (5a, b) it is found in case a) that $T_{sort} \sim n^2$ with good approximation. By application of more effective sorting techniques [8] we could achieve at best $T_{sort} \sim n \log_2(n)$ but even this affords a high expense of computertime for e.g. $n = 10^5$ or $n = 10^6$ trials.

The LRE-algorithm on the other hand keeps T_{sort} comparatively low. It can be shown in case b) that a

simple sorting technique with right-side input into memory FS resp. left-side input into GS in Fig. 4 is practically optimal because the contents of FS resp. GS represent sorted x-values of the left resp. right tail of the distribution function in each interim state $i = 1, 2, \ldots, N_F$ resp. N_G.

6. Final Remarks

6.1. The LRE-algorithm based on the considerations [4], [5] and [6] is not confined to the evaluation of simulated random data but represents a general method in statistics to obtain an a priori unknown distribution function $F(x)$ from measured data under control of an objective error measure. For example the large volume data obtained by traffic measurements in telephone or data networks [9], [10] could be evaluated favorably by means of the LRE-algorithm, provided that the independence requirement is fulfilled, see Section 6.3.

6.2. The LRE-algorithm can be developed further to perform a *window-mechanism:* given the parameters $F_{\min}$, $d_{F\max}$ we might be interested to observe the details of a certain segment F_1 to F_2 ($F_{\min} < F_1 < F_2 < 1$) of the curve $F_n(x)$ with higher accuracy, i.e. with a reduced maximum relative error $d^*_{F\max} < d_{F\max}$. This can be achieved if the evaluation program opens the window at F_1 by switching the integer r from the value eq. (7a) to the value

$$r = \text{ENTIER}\,(d^{*-2}_{F\max}) - 1 \qquad (9)$$

and closes it at F_2 by switching r back to eq. (7a). This concept can be generalized to allow for several windows along $F_n(x)$ resp. $G_n(x)$.

6.3. Simulation of and traffic measurement in queueing networks usually leads often to correlated i.e. *dependent* random values [11] and it is therefore of great practical importance to evaluate such correlated data by statistical methods which avoid "information loss" as it is associated e.g. with the batch mean method [2]. In [12] Bayes-methods have been applied to teletraffic birth death processes and it can be generally assumed that the objective statistics of Markov- resp. Semimarkov-processes and of certain elementary multiparameter problems such as given by the multinomial distribution will be needed to deal with many types of correlated random variables.

Appendix

A random process with two mixed E_k-distributions is described by the p.d.f.

$$f(x) = P_1\, a_1^{-1} \frac{(x/a_1)^{k_1-1}}{(k_1-1)!} \exp(-x/a_1) + \\ + (1-P_1)\, a_2^{-1} \frac{(x/a_2)^{k_2-1}}{(k_2-1)!} \exp(-x/a_2)\,. \qquad (10)$$

In order to gain a process with a rare event detail for the measurement Fig. 7 we have used the parameter set of the density $f(x)$ Fig. 9 where the left

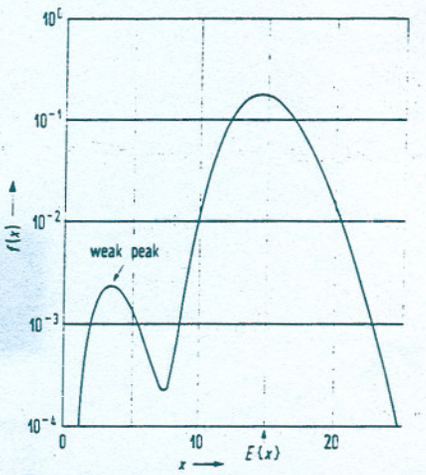

Fig. 9. Mixed E_8-, E_{45}-distribution: p.d.f. $f(x)$ eq. (10) with $a_1 = 1/2$, $k_1 = 8$, $a_2 = 1/3$, $k_2 = 45$, $P_1 = 0.008$.

peak is much weaker than the right peak (logarithmic scale!). In a previous paper the same parameter set has been used except $P_1 = 0.3$ yielding a density $f(x)$ with two peaks of comparable "strength", see Fig. 9 in [5]. For details of the software random generators see the appendices in [5] and [6].

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