Benefits and Limitations of Spatial Reuse in Wireless Mesh Networks

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ABSTRACT

Local area wireless networks are based on the cell topology: Clients associate to one of several access points, which are connected using a wired backbone. As high data rates are only available close to the access points, a dense infrastructure is needed. This results in high costs, especially for the installation of the wired backbone. A wireless mesh network can be used to reduce the deployment costs by connecting only few access points to the backbone; mesh nodes extend the coverage by forwarding data over wireless hops.

Since the wireless medium has to be shared by the nodes, multi-hop traffic requires a high capacity. Hence, mechanisms which increase the system capacity in wireless mesh networks are needed.

In this paper, we rate how much capacity can be gained by the introduction of spatial reuse. First, a system model of the wireless network is presented. This model includes a stochastic channel behavior and the signal strength/SINR requirements of a link. Additionally, the possibility of link adaption is incorporated. As the exact calculation of the system capacity using this model is NP-hard, we develop and survey heuristics that reduce the complexity.

Then, we apply the developed algorithms to evaluate different spatial reuse strategies. An upper bound is given by a network controlled by a omniscient scheduling entity; a lower bound is provided by refraining from spatial reuse. The results show that under the assumptions of the models at least a capacity increase by a factor of two is feasible; under optimal conditions a 12-fold increase is possible.

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1. INTRODUCTION

The recent development of ammendments for wireless networking standards, such as IEEE 802.11 "s" and 802.16 "j" considerably extends their deployment concepts. With the introduction of multihop configurations, it becomes possible to extend the coverage of a wireless network without costly and inflexible wired connections or increasing the transmission power.

With mesh networking, the coverage extension is provided using a hierarchical approach: The usually battery powered Stations (STAs) remain simple devices, without the ability to forward data for other participants. The mesh network is spawned solely among the Mesh Points (MPs), they relay the data as a transparent service for the STAs. Usually, one or more MPs are connected to a wired network and serve as a portal to the Internet.

A fundamental issue in wireless mesh networks is that performance degrades sharply as the number of hops traversed increases. This paper deals with the question how much capacity increase can be gained by exploiting spatial frequency reuse. This problem is not trivial since concurrent transmissions lead to an increase of the mutual interference, which requires reducing the transmission rates.

To assess the benefits of the optimal grade of spatial reuse, a model of the wireless channel has to be used which reflects the complex relations between concurrent transmissions, the resulting interference and the possibilities of rate adaption. Furthermore, it must be possible to use this model to calculate the resulting system capacity in different configurations of spatial reuse.

1.1 Related Work

The capacity of wireless communication networks is a popular research topic. Two major trends have evolved during the last years: i) To determine an upper bound by representing the capacity as a random variable and calculating its asymptotic properties, and ii) to compute the capacity of an arbitrary network using graph-theory and optimization algorithms.

1.1.1 Analytical Upper Bound

In their seminal paper [6], Gupta and Kumar exposed the limitations of multi-hop radio networks by computing the achievable throughput obtainable under optimal conditions by each node to $\Theta\left(W/\sqrt{n}\right)$, where n denotes the number of nodes and W the maximum transmission rate. Hence, the authors conclude that due to the vanishing throughput with large n, efforts should be targeted to small networks.

Several researchers have considered to extend the basic model, e. g. by incorporating different network structures[10] or mobility[5]. Due to the chosen approach, they have in common that they derive asymptotic scaling laws that describe the theoretical capacity in the model assumptions. Hence, one must be careful to apply them to an arbitrary network instance with small number of nodes and a given topology.

1.1.2 Graph-based capacity calculation

This disadvantage is addressed by several other researchers who concentrate on the calculation of capacity limits in a given network instance. In most of the works (e. g. [7, 8]), this is done by translating the properties of the wireless medium (reception probability, shadowing, interference) into two graphs: The connectivity graph $G = (V_G, E_G)$ and the conflict graph $C = (V_C = E_G, E_C)$, which expresses which links cannot transmit concurrently. After this translation, the capacity of the network is determined using graph-theory, e. g. node-coloring.

In the case of IEEE 802.11 and 802.16-based networks, the problem is even aggravated: The standard enables the transmitter to choose among a wide range of Modulation- and Coding Schemes (MCSs) for the transmission. If a highly sensitive MCS with many data bits per transmitted symbol is selected, the receiver is more susceptible to interference in comparison to a robust, low-rate MCS. Hence, it is impossible to build an interference graph without restricting each link to one MCS in advance. Therefore, many existing publications on exact capacity calculation restrict their link model to one MCS only, which ignores a characteristic feature of current standards.

1.1.3 Optimization-based capacity calculation

To our best knowledge, the only work which allows for different MCS and is still able to compute the capacity of a given network is presented in [15]. Here, the authors compare different transmission strategies by computing the set of achievable data rate combinations between all source-destination pairs in the network. *Basic Rates* are introduced as a key element, describing a set of active links in a wireless network at a given time. The challenge is to find the schedule of all feasible basic rates that minimizes the schedule's duration. The upper capacity bound can be extracted out of the duration of the optimal schedule and the amount of transported traffic.

Unfortunately, the computation of the upper capacity is NP-complete [2]. In consistence with this result, the number of basic rates and thus the runtime grows exponentially and renders the method unusable for networks of more than 20 nodes.

1.2 Our Contributions

Research on capacity calculations is often based on a simple system model: Characteristics of the network or the physical layer are neglected to avoid complex calculations.

To research the characteristics and benefits of spatial reuse and to judge the capacity gain which is related to it, the model needs to incorporate the conditions of the wireless channel and the receiver. Hence, we develop a system model which describes

- the stochastic nature of the wireless channel, including a correlated log-normal shadowing loss,
- the Signal to Interference plus Noise Ratio (SINR) and Received Signal Strength (RSS) requirements of the different transmission rates and
- the efficiency of the link depending on the selected MCS.

As a second distinction to existing work, we present crucial extensions of the method in [15] which allow for a calculation of the system capacity in much larger scenarios. New heuristics reduce the number of network states, resulting in acceptable computation times for reasonable sized networks. The evaluation of the heuristics shows a good approximation degree. Hence, to the best of our knowledge, this paper is the first concerned with the capacity evaluation of arbitrary, reasonable sized wireless networks using a realistic communication model.

Finally, we apply the developed algorithms to quantify the benefits of spatial reuse in mesh networks. The results allow for a sound judgment what capacity gains can be expected by improving existing Medium Access Control (MAC) protocols using concurrent transmissions.

2. SYSTEM MODEL

We consider a squared area A which is covered with a mesh network using m MPs. Each MP provides wireless Internet access to a subset of the n STAs which are distributed randomly in the area. In the following, the term *node* is used to denote both the STAs and the MPs if no differentiation is required.

The system model is partitioned into several submodels; each one describes the capabilities and the behavior of one layer in the ISO/OSI reference model: The channel submodel, the physical layer, the MAC and the network/routing layer.

2.1 Channel Submodel

The channel submodel determines the received signal quality of a transmission from node N_i to N_j, positioned at p_i and p_j, respectively. One of the most important requirements for this model is the inclusion of the severe shadowing which results from the combination of non line of sight conditions and the used spectrum, which is well above the 2 GHz band. This implies that not only the distance between the two nodes needs to be incorporated, but also their absolute positions in the scenario.

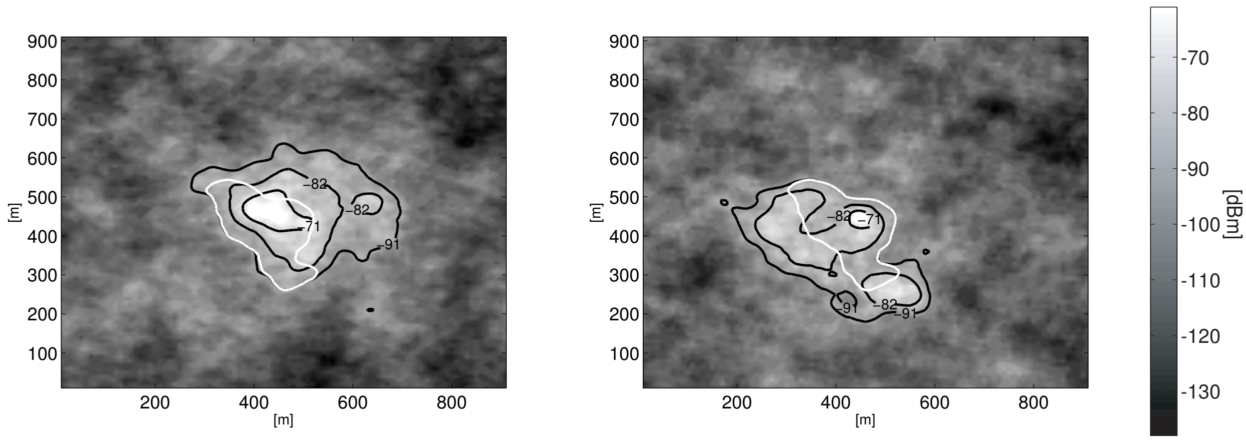
Consequently, in this model two factors attenuate the received signal: A deterministic path loss pl and a stochastic shadowing s. The first one is computed using the formula from [16]:

$$\begin{aligned}\mathrm{pl}(p_i, p_j)\,[\mathrm{dB}] =& 10\gamma \log_{10}\left(d(p_i, p_j)\right) \\ &+ 20 \cdot \log_{10}\left(\frac{c}{f_c \cdot 4\pi}\right),\end{aligned} \tag{1}$$

where

- γ stands for the path loss factor,
- $d(p_i, p_j)$ is the distance between p_i and p_j,
- c denotes the speed of light and
- f_c is the center frequency.

Measurements in urban city centers show that the fluctuation of the shadowing between two nodes can be characterized by a log-Normal distribution[9]. Furthermore, it



(a) Downlink (b) Uplink

Figure 1: Received Signal Strength (RSS), according to the channel parameters given in Section 4. The white line encloses the area where a bidirectional link is feasible.

was shown in [14], that spatial correlation properties of the random process play a significant role. Hence, we follow the discussion in [3, 17] and generate a 4-D log-Normal process that depends on the location of the transmitter and the receiver:

$$\mathrm{s}(p_i, p_j)\,[\mathrm{dB}] \sim N\left(0, \sigma_{\text{shadow}}\right).^{*} \tag{2}$$

The two-dimensional Joint Correlation Function (JCF) measures how fast the shadowing varies if the transmitter and the receiver are moved by $[\Delta_i, \Delta_j]$. As each movement has an independent and equal effect on the correlation[17], it can be decomposed into two independent identical one-dimensional Autocorrelation Functions (ACFs). This ACF can be modeled using an exponential decay function[4] that depends on the distance moved d:

$$R\left(d\right) = e^{\frac{-|d|}{d_{cor}} \ln 2}. \tag{3}$$

The parameter d_{cor} corresponds to the distance at which the correlation drops to 50%.

Together with optional antenna gains g_i and g_j at the transmitter and the receiver, the received signal strength during a transmission with transmission power P_i dBm can be computed as

$$\begin{aligned}\mathrm{P}\left(N_i, N_j\right)[\mathrm{dBm}] = \mathrm{P}_i + g_i + g_j \\ - \mathrm{pl}(p_i, p_j) - \mathrm{s}(p_i, p_j)\end{aligned} \tag{4}$$

An exemplary result of this model can be seen in Figure 1: It displays two RSS footprints for a node N which is positioned in the center. The RSS footprint in Figure 1a shows the reception power $\mathrm{P}\left(N, N_j\right)$ of all possible nodes N_j which are positioned in the area, whereas Figure 1b shows the reception power of node N, $\mathrm{P}\left(N_j, N\right)$, during a transmission of a node N_j.

Clearly, the node's transmission area is frayed and non-contiguous, which is a result of the significant influence of the shadowing in the spectrum under consideration. Hence, a transmission range which bounds the distance between the transmitter and the receiver cannot be defined. Furthermore, it is obvious that link symmetry cannot be assumed.

*If measured in dB, the shadowing process has a Normal probability distribution function

2.2 Physical Layer Submodel

The Physical Layer (PHY) submodel decides under which conditions a packet transmission is successful, i. e. the packet is decoded error-free at the receiver.

In our model, the success probability is calculated from two parameters: First, the RSS, determined by the channel model, must be large enough to allow for a correct identification of the signal at the receiver. Second, if concurrent transmissions are active, the mutual interference plays an important role. This is modeled by the SINR ratio.

Let $\{N_t : t \in T\}$ be the set of transmitting nodes at a given instance. If now node $N_j, j \notin T$ receives from node $N_i, i \in T$, the SINR at N_j is

$$\mathrm{SINR}(N_i, N_j, T) = \frac{\mathrm{P}\left(N_i, N_j\right)}{Noise + \sum_{k \in T, k \neq i} \mathrm{P}\left(N_k, N_j\right)} \tag{5}$$

For a successful packet transmission from N_i to N_j, a set of conditions have to be fulfilled.

- N_i must transmit only to node N_j, i. e. it cannot transmit and receive at the same time.
- N_j must receive only from node N_i, i. e. it cannot receive and transmit at the same time.
- The reception power $\mathrm{P}\left(N_i, N_j\right)$ must exceed a threshold $\mathrm{Thres}_{\mathrm{P}}$ (MCS).
- The SINR during the transmission must be above a threshold $\mathrm{Thres}_{\mathrm{SINR}}$(MCS).

Both threshold conditions are dependent on the MCS which is selected by the transmitter.

If all four conditions are fulfilled, the Packet Error Rate (PER), depending on the MCS, can be calculated, e. g. using the approach in [11] for IEEE 802.11a. The resulting transmission rate depends on the link efficiency, i. e. the fraction of time needed to transmit the data divided by the total transmission duration. It depends on the packet length (including header, data, checksum and padding), the waiting times, the length of the acknowledgment and the number of expected trials for a successful packet transmission.

2.3 The Medium Access Control Submodel

While the PHY submodel abstracts the behavior of a single link in the network, the Medium Access Control (MAC) submodel is concerned with the behavior of the complete network. Due to the multiple access nature of the wireless channel, transmissions have to be scheduled collision-free. In a real implementation, this functionally is implemented by protocol overhead, i. e. in the case of IEEE 802.11 by the exponential backoff.

To estimate the maximum achievable throughput capacity, we assume an omniscient and omnipotent coordination entity. This entity

- has full knowledge about the PHY submodel for each link,
- controls the traffic load on each link, so that the end-to-end requirements are met,
- generates a schedule for the transmissions and
- disseminates this schedule to the nodes without transmission costs.

All nodes operate under the guidance of this hypothetical controlling entity. This allows for an optimal schedule to be followed by the network, which maximizes the network capacity. How to find this optimal schedule under the restrictions and potentials of the system model is described in Section 3.

2.4 The Routing Submodel

Wireless mesh networks apply routing protocols known from mobile ad-hoc networks to determine a path between sources and sinks, possibly spawning over multiple hops. By incorporating information from the PHY and MAC into the path metrics, the characteristics of the wireless channels are recognized by the protocol.

Similar to the MAC submodel, we abstract the capability of the routing protocol by describing its effects on the traffic streams. While a STA associates to the MP with the highest RSS, path selection among the MP is driven by the end-to-end cost, measured in the total transmission duration. As it is already known from the PHY submodel which links can operate using a given rate, we can use the Floyd-Warshall algorithm to find the cheapest routes.

3. OPTIMIZATION: FINDING THE SHORTEST SCHEDULE

The system model formulates the restrictions and potentials under which the nodes in the mesh network have to operate. To find the capacity limit of the network under these constrains, we introduced a hypothetical entity that controls the operation of all nodes. The goal of this entity is to find the optimal schedule among all possible sequences of (concurrent) transmissions.

For simplicity, time is divided into intervals of 1 s length; in each interval the same schedule is applied to align the transmissions which result in the delivery of the traffic to the destinations. A *schedule* is defined as a list ((S_1, δ_1) ; (S_2, δ_2) ; …; (S_s, δ_s)) of network states S and respective durations δ; $\sum_{i=1\dots s} \delta_i$ denotes the duration of the schedule. A *network state* represents a possible activity in the network by enumerating the active links including the transmitter, the receiver, the MCS and the source and destination of the packet. An exemplary state would be

$$\mathrm{S} = \left\{ \begin{array}{ccccc} \text{source} & \text{tx} & \text{MCS} & \text{rx} & \text{destination} \\ (N_2 \rightsquigarrow) & N_2 & \rightarrow^{54\,\mathrm{Mb/s}} & N_3 & (\rightsquigarrow N_1) \\ (N_6 \rightsquigarrow) & N_5 & \rightarrow^{12\,\mathrm{Mb/s}} & N_4 & (\rightsquigarrow N_1) \end{array} \right\}$$

which describes two concurrent transmissions, one from node N_2 to node N_3 with $54\,\mathrm{Mb/s}$ with data originated at node N_2 and destined to node N_1, etc.

A network state is *feasible* if each transmission is successful according to the PHY model. A feasible schedule must contain feasible network states only; furthermore, it must fulfill all traffic requirements. Finally, a schedule is optimal if no other feasible schedule exists with a smaller duration.

The calculation of the optimal schedule is divided into two steps. In step one, the set of feasible network states is computed, denoted $\mathcal{S}$. This is done by several iterations: First, we start with the set $\mathcal{S}^1$ consisting of all feasible network states with one transmission. Then, we successively generate $\mathcal{S}^{c+1}$ by combining the network states out of $\mathcal{S}^c$ with the network states out of $\mathcal{S}^1$, if feasible. This results in the network states with $c+1$ concurrent transmissions. The process is repeated until the round c^* where no new feasible network states can be found; $\mathcal{S}$ is set to the union of all $\mathcal{S}^c$, $c = 1 \dots c^*$.

The second step uses the set $\mathcal{S}$ to compute the optimal scheduling. First, each found network state $\mathrm{S} \in \mathcal{S}$ is transformed into a matrix $\mathbf{s}$ with entries

$$\mathbf{s}[i,j] = \begin{cases} r, & \text{if node } N_j \text{ transmits data with r Mb/s} \\ & \quad \text{which origined at node } N_i, \\ -r & \text{if node } N_j \text{ receives data with r Mb/s} \\ & \quad \text{which origined at node } N_i, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Then, the traffic demand matrix T is generated in the same manner from the traffic requirements, enumerating the sources and the sinks instead of transmitters and receivers. Combined with the set $\mathcal{S}$ in matrix form, the optimization problem of finding the optimal schedule is translated into an instance of Linear Programming (LP):

$$\begin{aligned} \text{minimize } & f(\delta) = \sum_{i=1\dots|\mathcal{S}|} \delta_i \\ \text{such that } & \sum_{i=1\dots|\mathcal{S}|} \delta_i \cdot \mathbf{s}_i = T \\ & 0 \le \delta_i \le 1 \qquad i = 1 \dots |\mathcal{S}|. \end{aligned} \tag{7}$$

This task can be solved by any optimization toolbox, e. g. the one included in Matlab. The resulting output vector $\delta^* = (\delta_1^*, \dots, \delta_{|\mathcal{S}|}^*)$ assigns the durations to the network states. The resource utilization of the network which is needed to carry the traffic T is

$$\mathrm{RU}(\mathcal{S}, T) = f(\delta^*) = \sum_{i=1\dots|\mathcal{S}|} \delta_i^*. \tag{8}$$

As in [15], the *capacity region* $\mathcal{C}(\mathcal{S})$ is defined as the convex hull of all traffic settings which can be carried by the network in one time slot, i. e.

$$\mathcal{C}(\mathcal{S}) = \{T : \mathrm{RU}(\mathcal{S}, T) \le 1\mathrm{s}\}. \tag{9}$$

For the evaluation, we use the uniform system capacity which represents the point of the capacity region where all r routes induce the same traffic t into the network:

$$\mathcal{C}^u(\mathcal{S}) = \frac{r \cdot t}{\mathrm{RU}\,(\mathcal{S}, T)}. \tag{10}$$

The complexity of both parts of the algorithm (the creation of the network states and the solving of the LP instance) depends on the final number of network states $|\mathcal{S}|$, which increases exponentially with the number of nodes [15].

3.1 Heuristics

Due to the computational complexity, the described algorithm is restricted to networks with up to 20 nodes. Although this provides ground for some experiments, it does not suffice for meaningful results.

Therefore, heuristics have to be applied which reduce the computational complexity. We develop three different methods which tackle the main reason for the long running time: The exponential growth of the network states.

3.1.1 Early Cut (EC)

Intuitively, an increase of the number of concurrent transmissions results in an increase of the network performance. A huge benefit can be expected by allowing a spatial reuse with two concurrent transmissions in comparison to no spatial reuse at all, whereas the change from seven to eight transmissions might be negligible.

This insight is exploited by the Early Cut (EC), Algorithm 1, which stops the generation of network states with more concurrent transmissions if only a minor capacity increase can be expected. The stopping threshold t is set to 1% in our case.

Algorithm 1 Scheduling using the EC heuristic

```
Compute S^1 by selecting all possible links
S^2 = S^1 ∪ {all feasible network states with two tx}
Compute RU(S^1, T) and RU(S^2, T)
c = 2
while RU(S^c, T) < RU(S^{c−1}, T) − t do
    S^{c+1} = S^c
    for all ns ∈ S^c do
        S^{c+1} = S^{c+1} ∪ {ns' = ns ∪ l;  l ∈ S^1 ∧ ns' is feasible}
    end for
    Compute RU(S^{c+1}, T)
    c = c + 1
end while
```

3.1.2 Selective Growing (SG)

A second heuristic uses the observation that only few network states among the created ones are used by the LP in the optimal solution. Although it cannot be predicted in advance which network states are needed, a prior run of the LP with a small subset of states identifies the candidates. If the LP assigns only a negligible duration to a network state, it is not used in the next round to generate new ones with more concurrent transmissions.

The pseudo-code reads as in Algorithm 2. Here, only network states are enriched with additional concurrent transmissions which have a duration that is larger than some predefined threshold $t > 0$.

Algorithm 2 Scheduling using the SG heuristic

```
1: Compute S^1 by selecting all possible links
2: c = 1
3: while new network states have been created during the
   last round do
4:     Compute RU(S^c, T) and the solution vector δ*_c
5:     S^{c+1} = S^c
6:     for all ns ∈ S^c with δ(ns) > t do
7:         S^{c+1} = S^{c+1} ∪ {ns' = ns ∪ l;  l ∈ S^1 ∧
           ns' is feasible}
8:     end for
9:     c = c + 1
10: end while
```

3.1.3 Selective Growing/Delete

A further step to reduce the complexity in comparison to Selective Growing (SG) is to delete those network states which were assigned a duration smaller than a threshold $t > 0$. Thus, the initialization of $\mathcal{S}^{c+1}$ in the previous algorithm, line 5 becomes

$\mathcal{S}^{c+1} = \mathcal{S}^c|_{\delta_c^* > t}$

Hence, the relation $\mathcal{S}^{c-1} \subseteq \mathcal{S}^c$ does not hold any more.

4. SETTING OF PARAMETERS

With appropriate parameter settings the described system model and the optimization algorithm can be applied to several wireless network technologies, e. g. IEEE 802.11(a/b/g) or IEEE 802.16. In this paper, we restrict the evaluation to networks based on IEEE 802.11a, which implies the following settings.

4.1 Channel Submodel

The channel submodel parameters are fitted to numerous measurements of real networks in urban city centers, presented e. g. in [18]. We use the frequency band for IEEE 802.11a at f_c =5.5 GHz, which is license-free for outdoor usage with a transmission power of 30 dBm.

To incorporate the shadowing resulting from buildings, the path loss factor γ is set to 3.5 and the variance of the shadowing process σ_{shadow} is 8 dB.

To avoid edge effects, we use a wrap-around technique: Given the border length b of the square area, the distance between two points (x, y) and (r, s) is

$$\sqrt{\min(|x-r|, b-|x-r|)^2 + \min(|y-s|, b-|y-s|)^2} \tag{11}$$

4.2 Physical Layer Submodel

Many of the Physical Layer submodel parameters, like the values needed to compute the transmission overhead, can be obtained from the IEEE 802.11a standard. We assume a payload length of 1500 B, the resulting overhead per MCS can be found in Table 1. Furthermore, the minimum Received Signal Strength to decode the signal correctly by a common 802.11a device are displayed, as published in [1].

Using the results in [11] allows for computing the PER depending on the SINR and the selected MCS. Combined with the traffic overhead, it becomes possible relate the SINR level of one link to the achieved transmission rate.

Table 1: Transmission overhead and minimum RSS using IEEE 802.11a, payload length 1500 B

MCS	Gross Rate	Net Rate	Efficiency	RSS_{min}
BPSK 1/2	6 Mb/s	5.5 Mb/s	92.0 %	-91 dBm
BPSK 3/4	9 Mb/s	8.1 Mb/s	89.5 %	-85 dBm
QPSK 1/2	12 Mb/s	10.4 Mb/s	86.9 %	-83 dBm
QPSK 3/4	18 Mb/s	14.9 Mb/s	82.7 %	-81 dBm
16-QAM 1/2	24 Mb/s	18.8 Mb/s	78.4 %	-78 dBm
16-QAM 3/4	36 Mb/s	25.7 Mb/s	71.5 %	-74 dBm
64-QAM 2/3	48 Mb/s	31.4 Mb/s	65.4 %	-73 dBm
64-QAM 3/4	54 Mb/s	34.3 Mb/s	63.4 %	-73 dBm

5. EVALUATION

Based on the analytical system model, the optimization algorithm and the parameter settings, we perform Monte-Carlo simulations to evaluate the mean upper bound capacity $\overline{\mathcal{C}^u}$. For each sample the following process is required:

1. We generate a new instance of the shadowing process and place the MPs according to [13] such that they form a connected network and cover at least 95% of the area. One MP is positioned at the center of the square, it acts as the portal to the Internet.

2. Then, n STAs are distributed randomly and associated to the MPs with the highest RSS.

3. Using the routing submodel, the routes from the STA to the portal and back are determined.

4. Assuming a traffic load of 1 Mb/s per STA with 90% downlink and 10% uplink, we compute the required resource utilization with the optimization algorithm. During the computation, it is possible to restrict the number of concurrent transmissions and thus model different degrees of spatial reuse. Finally, the uniform system capacity for this network topology is derived from the resource utilization and the traffic load.

The instance shown in Figure 2 presents a typical outcome of this process: The mesh network, consisting of 32 MPs, covers the area and provides connectivity for 50 STAs which are distributed randomly. The resulting uniform system capacity $\mathcal{C}^u$ is computed as 12.6 Mb/s, which represents one sample for the evaluation.

The iterative generation of samples results in the mean value of the uniform system capacity, $\overline{\mathcal{C}^u}$. During the evaluation, this value represents the efficiency of one spatial reuse degree (as determined in step 4). Additionally, we indicate the confidence interval with using a 95% confidence level to support the validity of the Monte-Carlo simulations.

The computational effort to reach a solid mean value is immense, as the variance of the results requires many samples to reduce the confidence interval. Thus, we start the capacity computation by validating the presented heuristics to reduce the computational complexity. After having found an acceptable combination of heuristics we apply it to evaluate the benefits of spectrum sharing in wireless mesh networks.

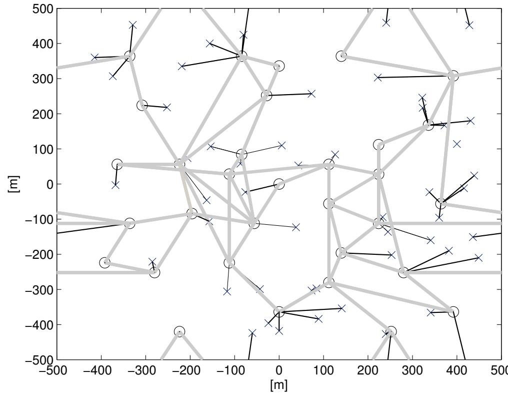


Figure 2: Exemplary scenario for the evaluation, consisting of the meshed MPs (circles) and 50 associated STAs (crosses).

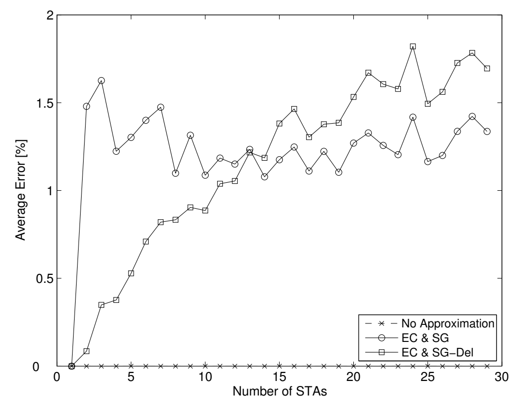


Figure 3: Relative error introduced by the heuristics

5.1 Validation of Heuristics

We have presented in Section 3.1 the heuristics Early Cut (EC), Selective Growing (SG) and Selective Growing/Delete (SG/Del). As they restrict the set of possible network states, the optimal schedule is only approximated. To assess the approximation error, the mean capacity computed by the exact algorithm is compared to the results of the heuristics in a scenario which is still feasible for the exact algorithm.

Figure 3 shows the mean error for the combination of the heuristics EC&SG and for EC&SG/Del. While EC&SG show a constant average error of about 1 to 1.5%, the average error for EC&SG/Del grows with the number of STAs. After about 20 STAs the growth rate decreases and remains below 2%, showing a similar fluctuation as the EC&SG error.

While the average computation time of the exact algorithm reaches one hour per sample for 30 nodes, both heuristics remain in the range of seconds. Considering the high accuracy of both schemes, and taking into account that envisaged networks contain more than 30 MPs and 100 STAs, leads to the conclusion that EC&SG/Del is suitable to compute the capacity limits. Hence, all further computations are based on this selection.

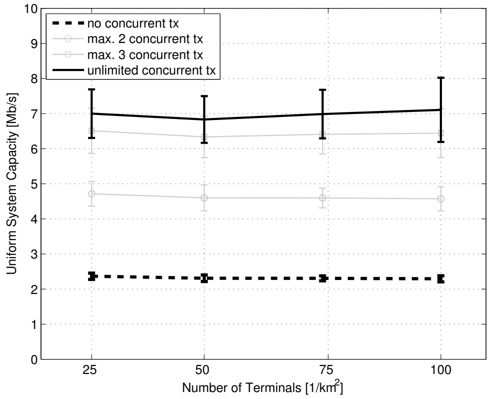


Figure 4: With the introduction of spatial reuse, the system capacity can grow by a factor of three compared to a mesh network where no concurrent transmissions are allowed.

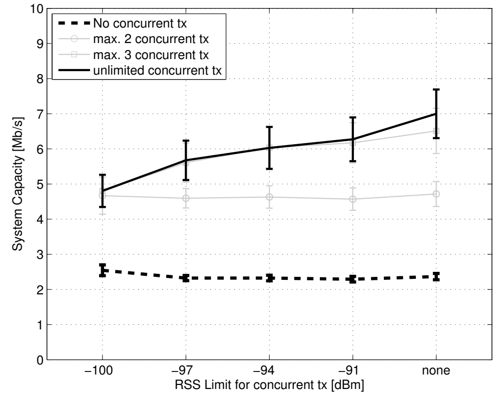


Figure 5: An RSS-Limit blocks transmissions in the vicinity of a link. Depending on the size of the blocked area, spatial reuse becomes less beneficial, but is still able to increase the capacity by a factor of two.

5.2 Capacity Increase by Spatial Reuse

Our first evaluation concerns the growth of the mean uniform system capacity $\overline{\mathcal{C}^u}$ which is obtained by the introduction of spatial reuse. As a reference value, the mean uniform system capacity without concurrent transmissions is used, which can be obtained easily by restricting the set of possible network states to $\mathcal{S}^1$, the states which consist of one transmission only.

The results for the sets $\mathcal{S}^c$ for $c = 2$ and 3 show how the system benefits from concurrent transmissions. Finally, $\overline{\mathcal{C}^u}\left(\mathcal{S}^{\text{inf}}\right)$ provides the mean upper bound for any spatial reuse concept using this system configuration.

The results can be seen in Figure 4. A first insight is gained from the constant relation of the system capacity and the number of terminals: Although the optimization space grows if more terminals are used, the bottleneck is still determined by the single portal MP. Hence, the system capacity does not change.

If more spatial reuse is allowed, the system capacity grows: It is doubled if two concurrent transmissions are allowed; the introduction of three transmissions results in an increase by 2.7; finally, optimal spatial reuses triples the capacity to $7\,{}^{\text{Mb}}/_{\text{s}}$.

5.3 Non-optimal Spatial Reuse Configuration

Although this optimal spatial reuse capacity is calculated using an existential argument (the applied schedules are known), it can hardly be reached using a distributed scheduling protocol. Hence, the question arises what capacity limits are obtained using a non-optimal, more realistic spatial reuse concept.

To model this, we abstract the RTS/CTS scheme from IEEE 802.11: Before a transmission takes place, the channel is reserved using two small indication messages, send by the transmitter and the receiver. Any node which receives this message refrains from accessing the medium. Hence, a noise-free area surrounding the link protects the transmission from unintended collisions, which facilitates the distributed scheduling of concurrent transmissions. Reservation-based MAC protocols extend the size of this zone to allow for planned concurrent transmissions[12].

Here, the abstraction of this mechanism is done by defining an RSS-Limit for concurrent transmissions: We assume that the transmitter and the receiver use a non-interference protocol to block any transmission up to a distance which is defined by the RSS-Limit. If this limit is very low, the distributed scheduling of transmissions becomes very simple; a high RSS-Limit results in many possibilities and the problem becomes very complex. The implementation of this abstraction into the optimization algorithm is very simple: During the generation of network states, new concurrent links are added only if they fulfill this additional requirement.

Since the influence of the number of STAs is low, Figure 5 shows the results for 100 STAs/km^2 only. The RSS limit is varied between -91 dBm (presenting the lowest RSS for a successful reception in the PHY submodel) and -100 dBm, which models a dissemination of future reservation in the (approx.) 3-hop neighborhood. Additionally, the values from Figure 4 for 100 STAs are repeated. We can see that this parameter significantly lowers the benefits of spatial reuse: The capacity gain is limited to a factor below two for the most demanding RSS setting.

Furthermore, it is no longer beneficial to target the optimal spatial reuse: The capacity upper bound is reached even if only up to three concurrent transmissions are allowed.

5.4 Directed Antennas and Spatial Reuse

In contrast to mobile ad-hoc networks, the topology of the mesh network is stationary during the operation. Hence, directed antennas can be mounted on the MPs and adjusted towards neighboring MPs. As a result, the affected link becomes much less susceptible to the inference of concurrent transmissions, which increases the possibilities of spatial reuse. In our model, we incorporate the benefits of directed antennas by the value of the receive antenna gain g_j in the computation of the reception power (Equation 4).

Figure 6 shows how the system capacity is increased by the introduction of either one or two directed antennas per MP, using an antenna gain of 5 dB to 20 dB. Considering the results without any concurrent transmissions (dashed lines), we can see that the capacity rises linearly with the antenna gain, up to the point where it can be tripled using

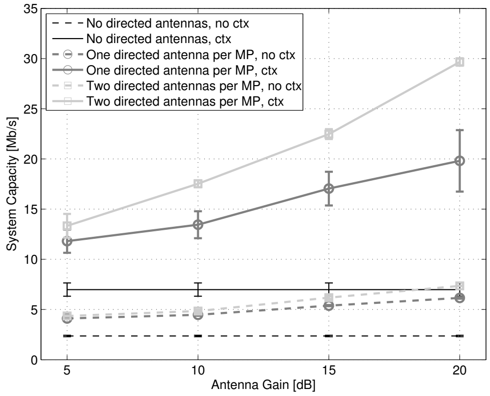


Figure 6: The usage of directed antennas for MP-links allows for more possibilities to align concurrent transmissions (ctx). Hence, the capacity can be increased by up to four times in comparison.

20 dB antennas. The difference between MPs with one or two antennas are negligible.

The combination of antenna gain and spatial reuse leads to a significant capacity increase. In the simplest case with one directed antenna, the increase is roughly the product of the gain by the spatial reuse and the directed antenna.

If two directed antennas can be used, more possibilities to align concurrent transmissions become available. Hence, the increase becomes higher than the product of the two influencing factors. E. g. with two 20 dB antennas alone, the capacity is tripled; the same holds for the introduction of spatial reuse. If both factors are combined, a the capacity is twelve times higher than the lower limit.

6. CONCLUSION

In this paper, we analyzed the relations between spatial reuse and the resulting system capacity in wireless mesh networks. A detailed system model is combined with an optimization algorithm to compute the capacity for arbitrary networks and a given traffic load. To allow for the computation of reasonable sized networks with more than 100 nodes, we develop the heuristics EC, SG and SG/Del.

By the application of the optimization algorithm in a Monte-Carlo fashion to random topologies and shadowing conditions, the expected system capacity is estimated for different spatial reuse strategies.

A lower bound is obtained in a network which does not use concurrent transmissions; the upper limit is given by the optimization algorithm without any restrictions. We can see that the introduction of two concurrent transmissions doubles the capacity, with three the upper bound is nearly reached in the scenarios under consideration.

Interference-avoiding protocols show similar promising results: Already with a RSS-limit of -94 dBm, 85% of the capacity limit is reached. Hence, one of the main conclusions of our paper is that a wireless mesh network must not target the maximum spatial reuse to reach a high system capacity.

The analysis of the combination of directed antennas and spatial reuse reveals further interesting results: While each technology alone is able to reach a capacity of about 3 times of the lower limit, the combination leads to a twelve-fold increase.

In our future work, the developed algorithms will be used to evaluate other interesting mechanisms in wireless mesh networks. Candidates are the introduction of transmit power control and the usage of multiple channels. Similar to this paper not only the upper bound capacity is of interest, but also how simple algorithms compare to this limit.

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